Orthogonal Basis

1. Let $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$. Find the coordinate vector for $\mathbf{u}$ in the basis $\mathcal{B}$ of the subspace W.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$$

2. Solve the following system of linear equations without doing any row reduction. (Hint: the columns of the coefficient matrix are orthogonal to each other.)

$$\begin{array}{rcrcrcr} x_1 & + & 6x_2 & + & 2x_3 & = & 23 \\ 2x_1 & - & x_2 & + & x_3 & = & 1 \\ 3x_1 & & & - & 16x_3 & = & -29 \\ 4x_1 & - & x_2 & + & 11x_3 & = & 23 \end{array}$$

Orthogonal Projections and Best Approximation

1. Let $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{y}$ be the vectors in $\mathbb{R}^3$ given below and let $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(a) Find the vector in W which is closest to $\mathbf{y}$.

(b) Find the distance from $\mathbf{y}$ to W.

(c) Find a vector $\widehat{\mathbf{y}}$ in W and a vector $\mathbf{z}$ orthogonal to W such that $\mathbf{y} = \widehat{\mathbf{y}} + \mathbf{z}$.