Dot Product and Friends

1. $$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix}$$

   (a) Find the lengths of $\mathbf{u}$ and $\mathbf{v}$—i.e. find $||\mathbf{u}||$ and $||\mathbf{v}||$.

   (b) Find the distance between $\mathbf{u}$ and $\mathbf{v}$.

   (c) Find $\mathbf{u} \cdot \mathbf{v}$.

   (d) Find $\mathbf{u} \cdot (\mathbf{u} + 2\mathbf{v})$

   (e) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (f) Find a nonzero vector which is orthogonal to $\mathbf{u}$

   (g) Find a unit vector in the same direction as $\mathbf{u}$.

2. Let $\mathbf{u}$ and $\mathbf{v}$ be as in the previous problem. Find a vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

3. True or false: three nonzero orthogonal vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.

4. True or false: If $\mathbf{u}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$ then it is orthogonal to $2\mathbf{v} + 3\mathbf{w}$.