Eigenvectors

1. Which of the following are eigenvectors of $\begin{bmatrix} 2 & 1 \\ -2 & 5 \end{bmatrix}$? For each one that is an eigenvector, state the corresponding eigenvalue.

   (a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} -3 \\ -6 \end{bmatrix}$

2. Suppose $\mathbf{v}_1$ and $\mathbf{v}_2$ are eigenvectors of A, both with eigenvalue 5. True or false: $3\mathbf{v}_1 - \mathbf{v}_2$ is an eigenvector of A. If true, find the corresponding eigenvalue. If false, give a counterexample.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be reflection across the line $x_1 = x_2$. Find an eigenvector of T.

4. Find an eigenvector of the following matrix. You should not need to do any calculation.

$$\begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 17 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

5. For both matrices below, find the eigenvalues and corresponding eigenspaces.

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$