The Matrix of a Linear Transformation Relative to a Basis

1. Let $T\colon M_{2\times 2} \to M_{2\times 2}$ be the linear transformation defined by $T(B) = AB$ where

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$$

   (a) Find bases for the domain and codomain of T and write the matrix of T relative to those bases.

   (b) Is T one-to-one? Onto?

   (c) Find a basis for the kernel and range of T.

Change of Basis

1. Suppose V is a 2 dimensional vector space and $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ and $\mathcal{C} = \{\mathbf{u}_1, \mathbf{u}_2\}$ are both bases for V. Also suppose that $\mathbf{u}_1 = \mathbf{v}_1 + \mathbf{v}_2$ and $\mathbf{u}_2 = \mathbf{v}_1 - \mathbf{v}_2$.

   (a) If $\mathbf{w}$ is a vector in V such that $[\mathbf{w}]_{\mathcal{C}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ then what is $[\mathbf{w}]_{\mathcal{B}}$?

   (b) Find a matrix A such that for all vectors $\mathbf{x} \in V$, $A[\mathbf{x}]_{\mathcal{C}} = [\mathbf{x}]_{\mathcal{B}}$.

   (c) Find a matrix B such that for all vectors $\mathbf{x} \in V$, $B[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$.

2. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$. Find the change-of-coordinates matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$.