Coordinates

1. Let's use linear algebra to solve the differential equation $y'' + 2y' - 5y = 3\sin(x) - 2\cos(x)$
   (a) Let V be the vector space $\text{span}\{\sin(x), \cos(x)\}$. Check that $\{\sin(x), \cos(x)\}$ is a basis for V.
   (b) Write the coordinate vector of $3\sin(x) - 2\cos(x)$ in the basis $\{\sin(x), \cos(x)\}$.
   (c) Let $T\colon V \to V$ be the linear transformation defined by $T(f) = \frac{d^2f}{dx^2} + 2\frac{df}{dx} - 5f$. Find the matrix for T in the basis $\{\sin(x), \cos(x)\}$.
   (d) Let A be the matrix you found in part (c) and $\mathbf{v}$ be the vector you found in part (b). Find a solution to $A\mathbf{x} = \mathbf{v}$.
   (e) Use your answer to part (d) to find a solution to the original differential equation.
2. Is $\{\sin^2(x), \cos^2(x), 1\}$ a basis for $\text{span}\{\sin^2(x), \cos^2(x), 1\}$?
3. Write the coordinate vector of the polynomial $p(x) = x^2 - 1$ in the basis $\{1, x, x^2 + x + 2\}$ for $\mathbb{P}_2$ (you don't need to check that this is a basis).
4. If the coordinate vector of a polynomial $p(x) \in \mathbb{P}_2$ in the basis $\{1, x, x^2 + x + 2\}$ is $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, what is $p(x)$?