Abstract Vector Spaces

1. Which of the following are vector spaces?
   
   (a) The set of sequences of real numbers that converge to 0.
   (b) The set of sequences of real numbers that converge to 1.
   (c) The set of letters in the Latin alphabet.
   (d) The set of $2 \times 3$ matrices in RREF.
   (e) The set of differentiable functions on the real numbers.

2. Are the polynomials $1, x^2, 3x^2 - 2$ linearly independent?

3. Is the sequence $(1, 0, 1, 0, \ldots)$ in the span of $(1, 1, 1, \ldots)$ and $(1, -1, 1, -1, \ldots)$?

4. Are the following $2 \times 2$ matrices linearly independent?

   \[
   \begin{bmatrix}
   1 & 0 \\
   0 & -1
   \end{bmatrix},
   \begin{bmatrix}
   0 & 0 \\
   1 & 0
   \end{bmatrix},
   \begin{bmatrix}
   0 & 1 \\
   0 & 0
   \end{bmatrix},
   \begin{bmatrix}
   -1 & -1 \\
   1 & 1
   \end{bmatrix}
   \]

5. Which of the following are linear transformations?

   (a) $T : \mathbb{P}_3 \to \mathbb{P}_3$ defined by $T(p) = \frac{dp}{dx}$.
   (b) $T : M_{2\times2} \to M_{2\times2}$ defined by $T(A) = A + I_2$.
   (c) $T : C([0,1]) \to \mathbb{R}$ defined by $T(f) = \int_0^1 f(x) \, dx$. 