(1) Find all eigenvalues and corresponding eigenspaces of the matrix \( A \).

\[
A = \begin{bmatrix}
-7 & -10 & -5 \\
5 & 8 & 5 \\
0 & 0 & -2
\end{bmatrix}
\]

(2) Suppose \( V \) is a vector space and \( \mathcal{B} = \{u_1, u_2, u_3\}, \mathcal{C} = \{v_1, v_2, v_3\} \) are two different bases for \( V \). If \( w = 3u_1 - u_2 + u_3 \) and the change of basis matrix from \( \mathcal{B} \) to \( \mathcal{C} \), \( P_{\mathcal{C} \leftarrow \mathcal{B}} \), is as given below, what is \( [w]_\mathcal{C} \)?

\[
P_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{bmatrix}
1 & 2 & 0 \\
2 & 0 & 0 \\
3 & 1 & 1
\end{bmatrix}
\]

Date: October 13, 2016.