MATH 54, FALL 2016, QUIZ 6

(1) Does the set $\{3 + 5x + 7x^2, 6 - 3x + 2x^2, 3 + 18x + 19x^2\}$ form a basis for $\mathbb{P}_2$?

(2) Let V be the set of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ that is spanned by $\mathcal{B} = \{\sin(x), \cos(x)\}$. Let $\mathcal{C} = \{1, x, x^2\}$ be the usual basis for $\mathbb{P}_2$. Let $T : V \to \mathbb{P}_2$ be the function defined by $T(f) = f(0) + f(\pi/2)x + f(\pi)x^2$.

(a) Write the matrix for T in terms of the bases $\mathcal{B}$ and $\mathcal{C}$. You do not need to check that $\mathcal{B}$ and $\mathcal{C}$ are bases or that T is a linear transformation.

(b) What is the kernel of T?

Note: $\sin(0) = \sin(\pi) = \cos(\pi/2) = 0$, $\cos(0) = \sin(\pi/2) = 1$, and $\cos(\pi) = -1$.