Math 10B, Quiz 3

1. (12 points) Suppose 200 students in math 10B this semester were all born in the same year. Show that there are at least two students who either have the same birthday or were born on consecutive days.

2. (1 point) For all n and all $k \leq n$,

$$\binom{n}{k}\binom{n+1}{k-1} = \binom{n}{k-1}\binom{n+1}{k}.$$

◯ True ◯ False

3. (1 point) For all n, $m \leq n$ and $k \leq m$,

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$

◯ True ◯ False

4. (1 point) On an exam, a question asks "How many 6 card hands can you form if the hand has to include at least one spade?" One student gives the answer $13\binom{51}{5}$, reasoning that they need to choose at least one spade (and there are 13 spades to choose from) and then they can choose any 5 of the 51 cards that haven't been used yet. The student is:

◯ Undercounting (i.e. their answer is too small)

◯ Correct

◯ Overcounting (i.e. their answer is too large)