Dynamics Worksheet 1

1. Suppose you roll two four-sided dice 100 times each and each time calculate the product of the two rolls. True or false: if you want to perform a $\chi^2$ test to check if the dice are fair based on the data you recorded then the degrees of freedom is 3.

2. For each of the following, verify whether the given formula is a solution to the given recurrence relation.
   
   (a) Recurrence relation: 
   
   $a_n = \sqrt{a_{n-1}^2 + 1}; a_0 = \sqrt{5}$
   
   Formula: 
   
   $a_n = \sqrt{n + 5}$

   (b) Recurrence relation: 
   
   $a_n = 2^{a_{n-1}} + a_{n-2}; a_0 = 1, a_1 = 2$
   
   Formula: 
   
   $a_n = \sqrt{n + 5}$

   (c) Recurrence relation: 
   
   $a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$
   
   Formula: 
   
   $a_n = n^2 + 3n$

   (d) Recurrence relation: 
   
   $a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$
   
   Formula: 
   
   $a_n = 3^n$

   (e) Recurrence relation: 
   
   $a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$
   
   Formula: 
   
   $a_n = 3^n - (-1)^n$

3. Find a formula for the $n^\text{th}$ Fibonacci number. Recall that the first two Fibonacci numbers (i.e. the 0th and 1st Fibonacci numbers) are both 1 and that to get the next Fibonacci number, you add the previous two. First formulate this as a recurrence relation and then try to solve it.

4. Find a solution to the following recurrence relation:

   $a_n = n \cdot a_{n-1}; a_0 = 1$

5. Challenge Question: Look up the rules to the Tower of Hanoi game and find a formula for the least number of moves it takes to win the game when there are $n$ disks.