Solve the following differential equations.

1. \( y'' + 7y' + 10y = 0 \) with initial conditions \( y(0) = 1 \) and \( y'(0) = 1 \).

   This is a linear, homogeneous equation with constant coefficients. So we can use the characteristic polynomial method. The characteristic polynomial is \( \lambda^2 + 7\lambda + 10 = (\lambda + 2)(\lambda + 5) \).
   
The roots are \(-2\) and \(-5\) so the general solution to the differential equation is \( y(t) = C_1 e^{-2t} + C_2 e^{-5t} \).
   
   Using the initial conditions to solve for \( C_1 \) and \( C_2 \), we find that
   
   \[
   \begin{align*}
   1 &= y(0) = C_1 e^{-2\cdot0} + C_2 e^{-5\cdot0} = C_1 + C_2 \\
   1 &= y'(0) = -2C_1 e^{-2\cdot0} - 5C_2 e^{-5\cdot0} = -2C_1 - 5C_2
   \end{align*}
   \]
   
   Solving this system of linear equations gives us \( C_1 = 2 \) and \( C_2 = -1 \). Therefore
   
   the final solution is \( y(t) = 2e^{-2t} - e^{-5t} \).

2. \( ty' - 4y = t^2 \)

   For this equation, we can use the integrating factor method. First we divide by \( t \) to isolate \( y' \). This gives us
   
   \[
   y' - \frac{4}{t} y = t.
   \]
   
   The integrating factor is
   
   \[
   e^{\int -\frac{4}{t} dt} = e^{-4\ln|t|} = e^{\ln|t|^{-4}} = |t|^{-4} = t^{-4}.
   \]
   
   Therefore
   
   \[
   t^{-4} y(t) = \int t^{-4} t dt = \int t^{-3} dt = -\frac{1}{2t^2} + C.
   \]
   
   Solving for \( y(t) \) we get a final solution of
   
   \[
   y(t) = -\frac{t^2}{2} + Ct^4.
   \]

3. \( t^2 y' = -y^2 \)

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This equation is separable. So we have

\[ \int -\frac{1}{y^2} \, dy = \int \frac{1}{t^2} \, dt. \]

Therefore

\[ \frac{1}{y} = -\frac{1}{t} + C. \]

Solving for \( y(t) \) gives us a final solution of

\[ y(t) = \frac{1}{-\frac{1}{t} + C}. \]

By the way, this is not equal to \(-t + C\) or to \(-t + \frac{1}{C}\).