Math 10B Probability Worksheet 4

1. There are n people who each have their own hat. You take all the hats and randomly rearrange them. Let the random variable X be the number of people who get their own hat back. What is $E[X]$?

2. Consider the scenario described in problem (1) when there are just two people. What is $\text{Var}[X]$?

3. If X is a random variable and $\text{Var}[X] = 0$, what can you say about X?

4. Suppose X is a nonnegative random variable and a is a positive number. Show that $P(X \geq a) \leq \frac{E[X]}{a}$.

5. Challenge Question: Show that if X is a random variable with $E[X] = \mu$ and $\text{Var}[X] = \sigma^2$ then for any $k > 0$, $P(|X - \mu| > k\sigma) \leq \frac{1}{k^2}$. [Hint: use the result of the previous problem applied to the random variable $(X - \mu)^2$.]

6. Suppose you roll 20 fair 6-sided dice. Let the random variable X be the sum of the rolls.

   (a) What is $E[X]$?

   (b) What is $\text{Var}[X]$?