120A Midterm 1 Instructions

October 5, 2011

Here are some previous exam problems. I've generally given three or four of these types of questions on a midterm. There are also similar questions on the hwk and in the book that I suggest that you look at carefully. You are also required to be able to characterize various curves by their curvature and torsion properties as explained in the book: lines, circles, planar curves, and spherical curves (Note that e_1, e_2, e_3 are spherical curves.)

For the midterm you are allowed to bring ONE sheet of paper with your notes. NO books or electronic devises are allowed.

From the book you should go through

Chapter 2: 1,3,4,5,6,9,10,16,17,18,19,20,25.

From the Pressley exercises you can also look at

1.2:3,4 and 2.2:5 and 2.3:2,3,4,12,19.

In addition I've isolated suitable old exam question below.

I'll have extended office hrs before the midterm: Monday 1-3pm, Tuesday Noon-2pm

1. Let $c(t) : I \to \mathbb{R}^3$ be a regular curve with speed $\frac{ds}{dt} = \left|\frac{dc}{dt}\right|$, where s is the arclength parameter. Prove that

$$\kappa = \frac{\sqrt{\frac{d^2c}{dt^2} \cdot \frac{d^2c}{dt^2} - \left(\frac{d^2s}{dt^2}\right)^2}}{\left(\frac{ds}{dt}\right)^2}$$

2. Let $c(t) : I \to \mathbb{R}^3$ be a regular curve such that its unit tangent field $e_1(t)$ is also regular. Let s be the arclength parameter for c and θ the arclength parameter for e_1. Show that

$$\kappa = \frac{d\theta}{ds}$$

and

$$\det\left(e_1, \frac{de_1}{d\theta}, \frac{d^2e_1}{d\theta^2}\right) = \frac{\tau}{\kappa}.$$

3. Let $c(t)$ be a regular curve in $\mathbb{R}^3$ with $\kappa > 0$. Prove that c is planar if and only if the triple product

$$\left[\frac{dc}{dt}, \frac{d^2c}{dt^2}, \frac{d^3c}{dt^3}\right] \equiv 0$$

4. Let $c(s) = (x(s), y(s))$ be a planar unit speed curve. Show that the signed curvature can be computed by

$$\kappa = \det[c', c'']$$

5. Let $c(s)$ be a unit speed curve in $\mathbb{R}^3$ Prove that

$$\det[c', c'', c'''] = \kappa^2 \tau.$$

   It is also possible to find formulas for

$$\det[c'', c''', c'''']$$

   etc.

6. Let $c(t) : I \to \mathbb{R}^3$ be a regular curve with positive curvature. Show that the unit tangent $e_1(t)$ is a regular and that, if θ is an arclength parameter for e_1, then

$$\begin{aligned} \frac{dc}{d\theta} &= \frac{1}{\kappa} e_1 \\ \frac{de_1}{d\theta} &= e_2 \\ \frac{de_2}{d\theta} &= -e_1 + \frac{\tau}{\kappa} e_3 \\ \frac{de_3}{d\theta} &= -\frac{\tau}{\kappa} e_2 \end{aligned}$$

7. Let $c(t) : I \to \mathbb{R}^3$ be a regular curve with positive curvature. Show that c lies in a plane if and only if the torsion vanishes.