120A Hwk 8

November 22, 2011

Homework problems due on **Wednesday Nov. 30, 2011**.
Do the following exercises:

1. Show that a generalized cylinder is minimal if and only if it is planar.
2. Show that a generalized cone is minimal if and only if it is planar.
3. Show that a surface given by $z = f(x) + g(y)$ is a minimal if and only if
$$\frac{\frac{d^2 f}{dx^2}}{1 + \left(\frac{df}{dx}\right)^2} + \frac{\frac{d^2 g}{dy^2}}{1 + \left(\frac{dg}{dy}\right)^2} = 0$$
4. Show that the surface given by $\sin z = \sinh x \sinh y$ is a minimal surface.
5. Show that the surface defined by
$$x \sin \frac{z}{a} = y \cos \frac{z}{a}$$
is a minimal surface.
6. Let $f(u, v) : U \to \mathbb{R}^3$ be a parametrized surface and $f^\epsilon(u, v) = f(u, v) + \epsilon\nu(u, v)$ the parallel surface. If f is minimal, then the principal curvatures for f^ϵ satisfy that
$$\frac{1}{\kappa_1^\epsilon} + \frac{1}{\kappa_2^\epsilon} = \text{constant}.$$
7. Let $f(u, v) : U \to \mathbb{R}^3$ be a parametrized surface such that f and its Gauss map $\nu(u, v) : U \to S^2$ are conformally equivalent. Show that either f is minimal or parametrizing part of a sphere. Hint: Show that the surface is umbilic at any point where the mean curvature is not zero. Then use our characterization of surfaces which are totally umbilic.
8. Let $f(u, v) : U \to \mathbb{R}^3$ be a parametrized surface. Show that
$$\frac{\partial\nu}{\partial u} \times \frac{\partial\nu}{\partial v} = K \frac{\partial f}{\partial u} \times \frac{\partial f}{\partial v}$$