120A Hwk 4

October 27, 2011

Homework problems due on Wednesday Nov. 2, 2011.

1. Compute the first fundamental form of the *Catenoid* (it is the graph of a catenary rotated around the z-axis)

$$f_0(\theta, t) = (\cosh t \cos\theta, \cosh t \sin\theta, t)$$

2. Compute the first fundamental form of the *Helicoid* (it is the surface traced out by a line that is rotated as it moves up the z-axis along a helix)

$$f_1(\theta, u) = (u\cos\theta, u\sin\theta, \theta)$$

then reparametrize this surface using $u = \sinh t$ and show that it has the same first fundamental form as the catenoid.

3. Reparametrize the catenoid using $t = t(u)$ so that it becomes equiareal, i.e., $EG - F^2 = 1$, where E, F, G are with respect to the new parametrization (u, θ).

4. Show that a graph parametrization $(u, v, h(u, v))$ is equiareal if and only if the function h is constant.

5. Let $c(s) : I \to \mathbb{R}^3$ be a Frenet curve parametrized by arclength. Define the *tangent developable* by

$$f(s,t) = c(s) + te_1(s)$$

Show that this gives a regular parametrization of a surface when $t \neq 0$. Show that

$$\begin{aligned} E &= 1 + t^2\kappa^2(s) \\ F &= 1 \\ G &= 1 \end{aligned}$$

6. Let $c(s) : I \to \mathbb{R}^3$ be a regular curve and $a \in \mathbb{R}^3$ a point not on the curve. Find a way of parametrizing the *generalized cone* that consists of all lines through a and $c(s)$ for $s \in I$. Show that this is a regular parametrization along as the curve isn't tangent to the line from a to $c(s)$.

7. Let $c(s) : I \to \mathbb{R}^3$ be a regular curve and $v \in \mathbb{R}^3$ a vector. Find a parametrization of the *generalized cylinder* that consists of the lines through $c(s)$ that are tangent to v. Show that this is a regular parametrization along as $c(s)$ is not tangent to v.

8. Show that all generalized cylinders have Cartesian parametrizations, by showing that one can parametrize the generalized cylinder using a planar curve that is contained in the plane perpendicular to v.