120A Hwk 4

October 20, 2011

Homework problems due on Wednesday Oct 26, 2011.

1. Let $\gamma(\theta)$ be a simple closed planar curve with $\kappa > 0$ parametrized by θ, where θ is defined as the arclength parameter of the unit tanget field e_1. Further assume that the width

$$w = \langle e_2(\theta), (\gamma(\theta + \pi) - \gamma(\theta)) \rangle$$

is constant. Show that:

$$w = \frac{1}{\kappa(\theta)} + \frac{1}{\kappa(\theta + \pi)}.$$

Start by establishing the facts:

$$\begin{aligned} \frac{d\gamma}{d\theta} &= \frac{1}{\kappa} e_1 \\ \frac{de_1}{d\theta} &= e_2 \\ \frac{de_2}{d\theta} &= -e_1 \\ e_1(\theta + \pi) &= -e_1(\theta) \end{aligned}$$

2. Let $\alpha(s)$ unit speed curve with $\kappa > 0$. Let θ be the arclength parameter for $\frac{d\alpha}{ds}$. Show that the curvature satisfies:

$$\kappa = \frac{d\theta}{ds}$$

3. Prove that if $\alpha(s)$ is an oval (a closed planar curve with positive curvature and no self intersections), then the unit tangent field e_1 is parallel to e_1'' at four or more points.

4. Prove that the concept of a vertex for a planar curve does not depend on the parametrization.

5. Let $c(t)$ be a closed Frenet curve in $\mathbb{R}^3$. Show that if its curvature is $\leq R^{-1}$, then its length is $\geq 2\pi R$.