LARGE SIMPLE CYCLES
IN
SYMPLECTIC COMPLEXES

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Based on a joint work with
Roy Meshulam, Ilan Newman
K : The i-faces of K
d-cycle: all (d-i)-faces have even degree.
Perform a walk on vertices of graph $G$ to a previously visited vertex without ever returning.

$|E|$ < $2k(N-1)$

Erdős–Gallai's Th. in graphs

Proof: (assuming degs $< k$)
density

size of max. simple d-cycle

of some function of $K_{p,1}$

$G(K) \leq \Omega$?

K: Simplicial complex. Show:

size of max. cycle

$E - G$ says $C(G) \geq \frac{|E|}{|V|} \cdot \nu(G)$.

Our goal
Mathieu's theorem appears in Nash-Williams' thesis. A subset of edges with maximal acyclic size of \((\mathcal{G})\) and further \(c(\mathcal{G}) \geq \frac{\text{max}_{e \in \mathcal{E}} \text{rank}(e)}{2\text{max}_{e \in \mathcal{E}} \text{rank}(e)}\). Instead of \(c(\mathcal{G}) \geq \frac{\text{max}_{e \in \mathcal{E}} \text{rank}(e)}{2\text{max}_{e \in \mathcal{E}} \text{rank}(e)}\), write \(c(\mathcal{G}) \geq \frac{\frac{1}{2} \text{ln}(1 - \frac{1}{c(\mathcal{G})})}{\text{density}(\mathcal{G})}\). What are minors of simplicial complexes? The set of all minors \(\{\mathcal{G} \mid c(\mathcal{G}) \leq k\}\) is closed under minors. Other ideas for a proof?
\( \text{Goal: Lower-bound } c(M) \text{ in terms of } \lambda(M) \)

\[ \lambda(M) = \text{max } E_1 \in \text{E}_1 \text{ rank}(E_1) \]

\[ c(M) = \text{max } \text{size circuit in } M \]

- Simplicial matroids: \( \mathcal{M}(K) \) without circuits 0
- Simplicial complexes: \( \mathcal{M}(K) \)
  - \( I \): Acyclic subsets of \( K(r) \)
  - \( I_\text{acyclic subset} \) suggests
  - \( I_\text{linear} \): \( I \subseteq \mathcal{E}(G) \)
  - \( I_\text{independent set} \) suggests
  - \( I_\text{closed under containment} \)
  - \( M = (\mathcal{E}, X) \) elements independent 0

- Graphs: \( G \)
- Linear matroid: \( \mathcal{E}(G) \)
Then, \( c(M/C) \leq c(M) = C \).

Let \( M \) be loopless and connected, \( C \) max. circuit in \( M \).

Seymour Lemma:

\[
\text{too weak, too heavy...}
\]

\[
\frac{c(M)}{\log^2 \log^2 \log^2 \text{size } M} < \frac{c(M)}{\log^2 \log^2 \log^2 \text{size } M}
\]

\[
\begin{align*}
\gamma(M) & \leq 2^{23k} \\
& \Leftrightarrow \text{graphic minor} = \text{linear (if) matroid lacks a size } k \\
& \text{Matroidal Tools [Gillen]}
\end{align*}
\]
A circuit in $M/A$

extend to ind. sets of $M/A$

ind. sets of $M/A$

contraction: $M/A \setminus A$

deletion: $M/A \setminus A \setminus E(M)$

$ \notin E(M)$ *

components

biclique

in graphs

equivalent classes: components

$\sim$ is an equivalence relation:

$e \sim f$ if both lie in some circuit.

Matroids: Connectedness and Minors
After the removal of the loops:

* The children of $x$ are the components of $M_x/C$

* The root has $M/C$

* $C_x$: max circuit in $M$

* $M_x$: a minor of $M$ has associated $M_x/C$

Every vertex $x$ in $M$ has a vertex of $M$.

$M$: connected, loopless

Decomposition tree of $M$
Let \( S_t(c) = \max \{ 1, E(c), \text{rank}(E) \} \). The Mathematical Theorem

\[ \text{Theorem:} \quad \text{If} \quad S_t(c) = S_t(c_{m+1}) \quad \text{then} \quad \text{rank}(E) = 1. \]
With further effort, and more simplicial complexes, we have:

$$G_d(K) = \sqrt{\frac{2}{d+1}} \mathcal{L}_d(K) - 1$$

Theorem: Let $A$ be a set of $d$-simplices on $[m]$. The extremal $A$ is shifted, and ranked $(A) \geq \left\lfloor \frac{n}{d+1} \right\rfloor$. Back to simplicial complexes
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