Percolation is Odd

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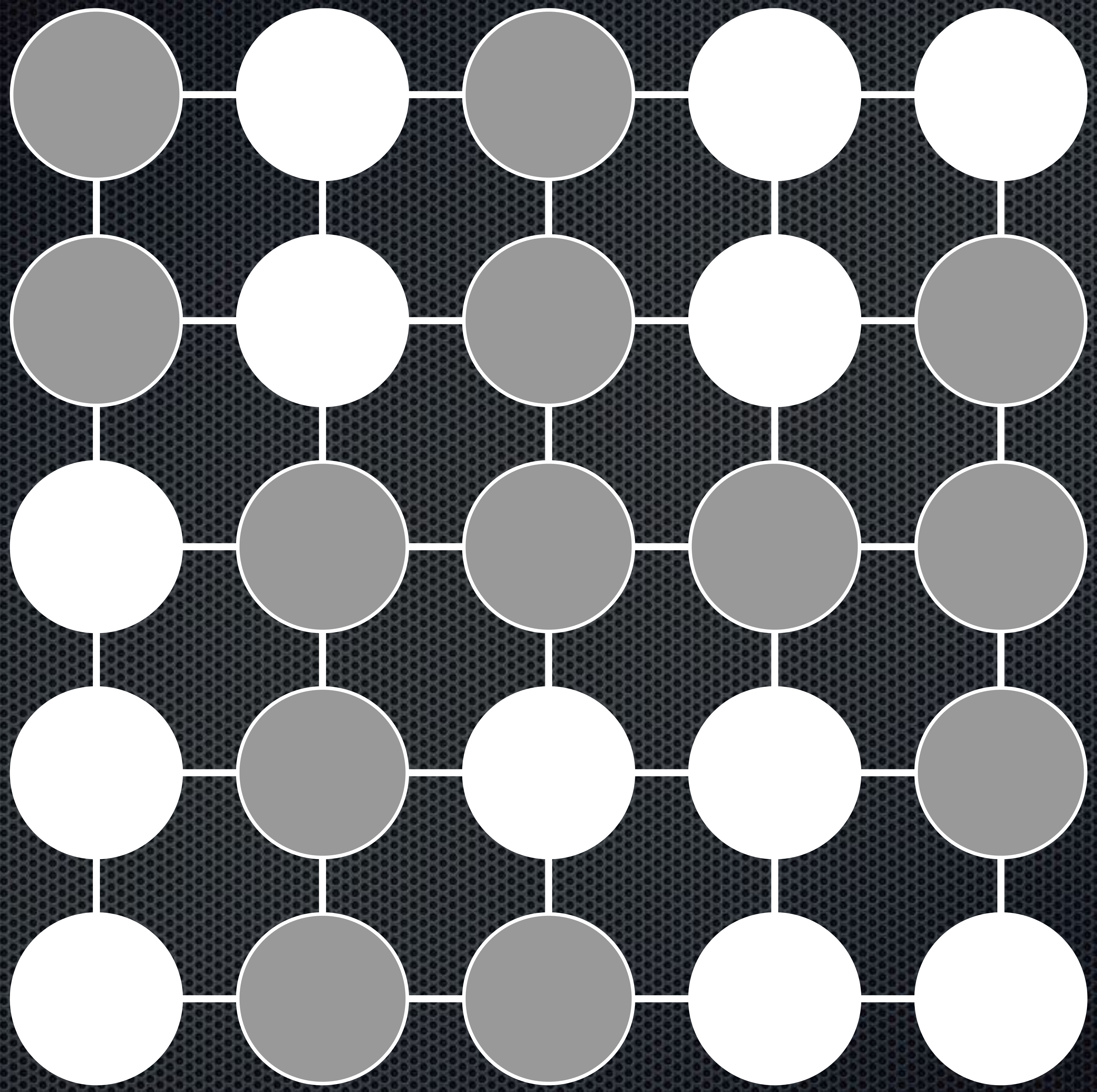
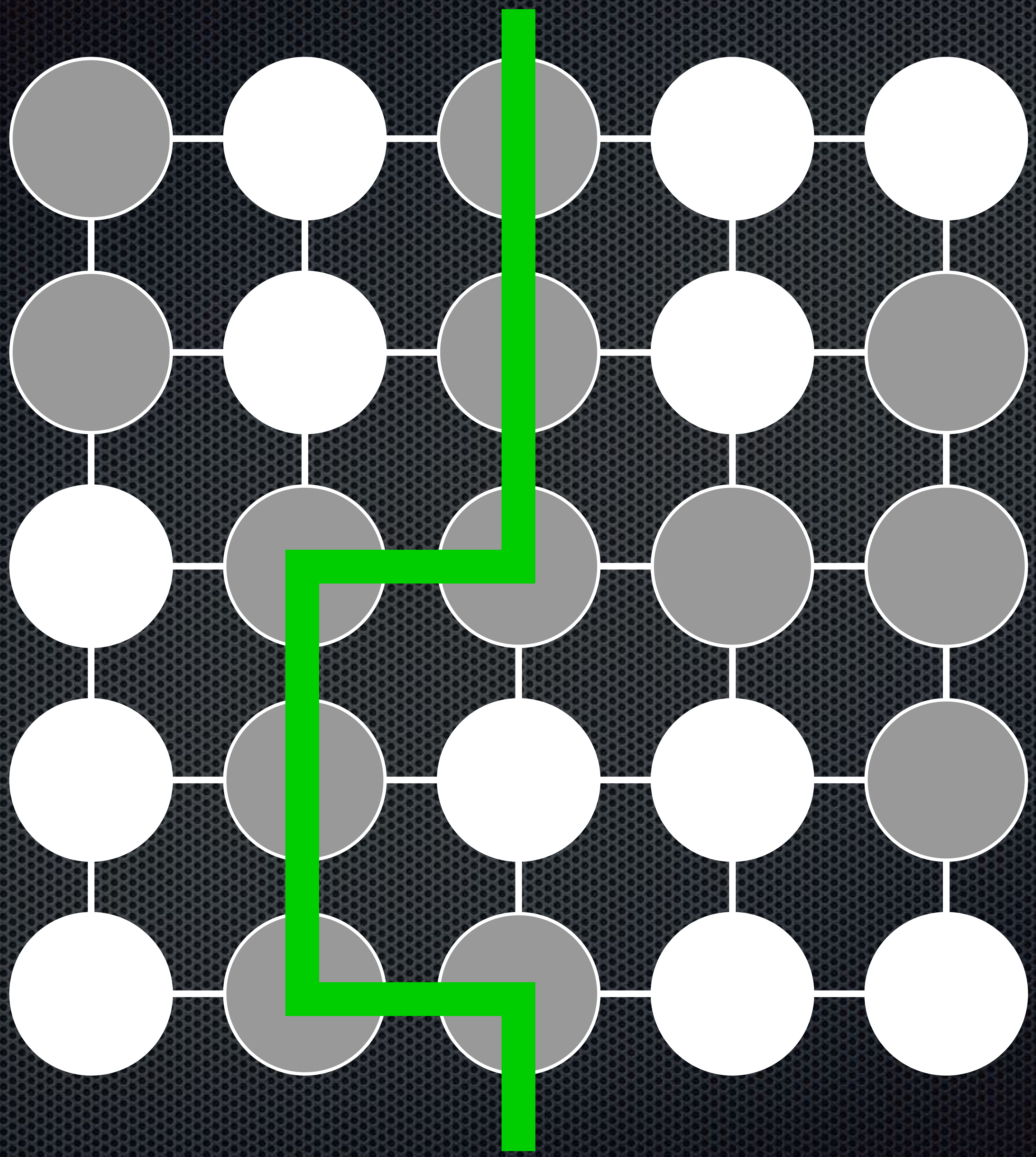




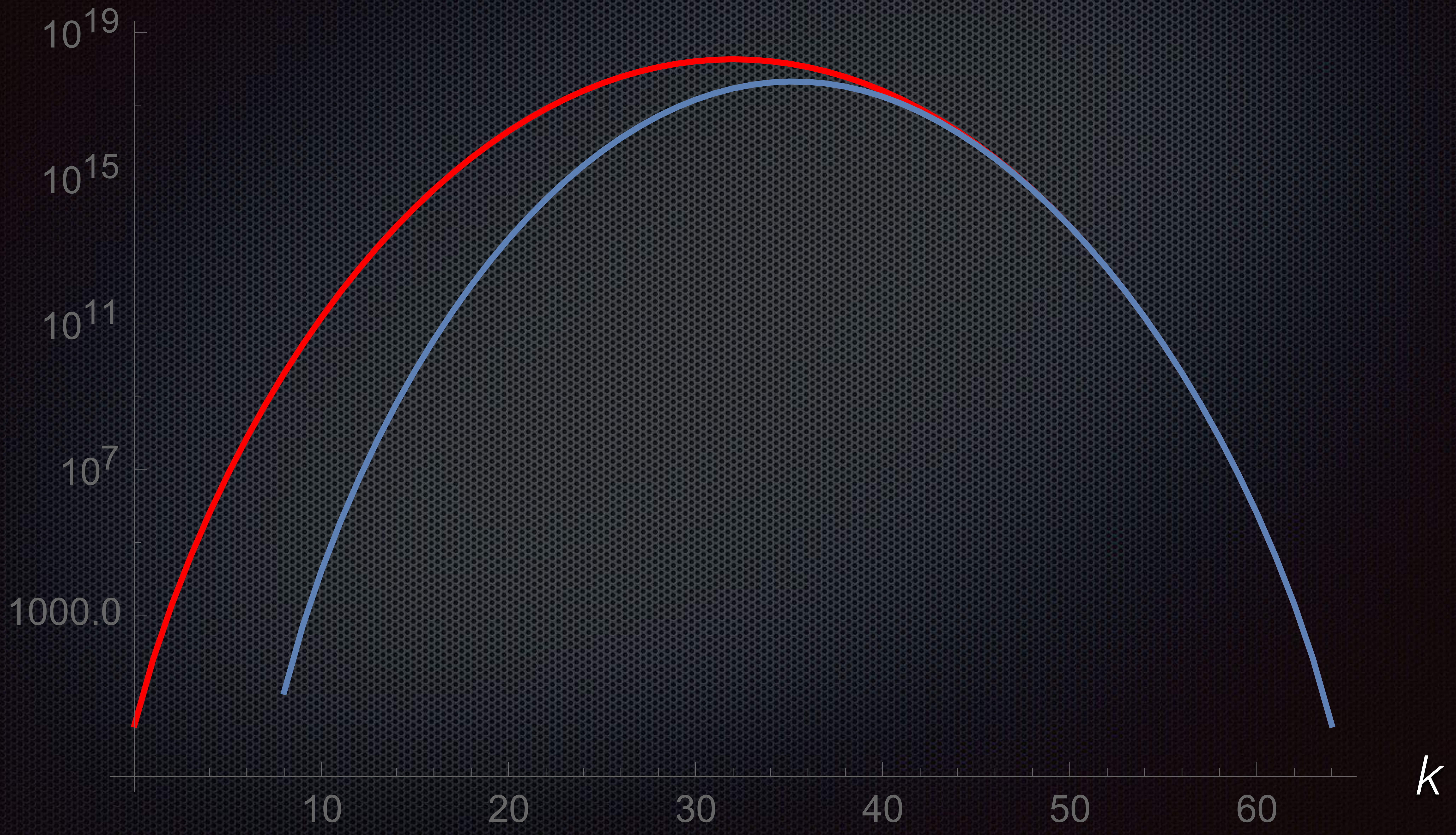
The Total Number of Spanning Configurations is Always Odd

Height

Width

	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	3	7	17	41	99	239	577
3	7	37	197	1041	5503	29089	153769
4	15	175	1985	22193	247759	2764991	30856705
5	31	781	18621	433809	10056959	232824241	5388274121
6	63	3367	167337	8057905	384479935	18287614751	868972410929
7	127	14197	1461797	144769425	14142942975	1374273318721	133267613878665

$A_{n,m}(k)$: # configurations with k occupied sites



$$R_{n,m}(z) = \sum_{k=0}^{nm} z^k A_{n,m}(k)$$

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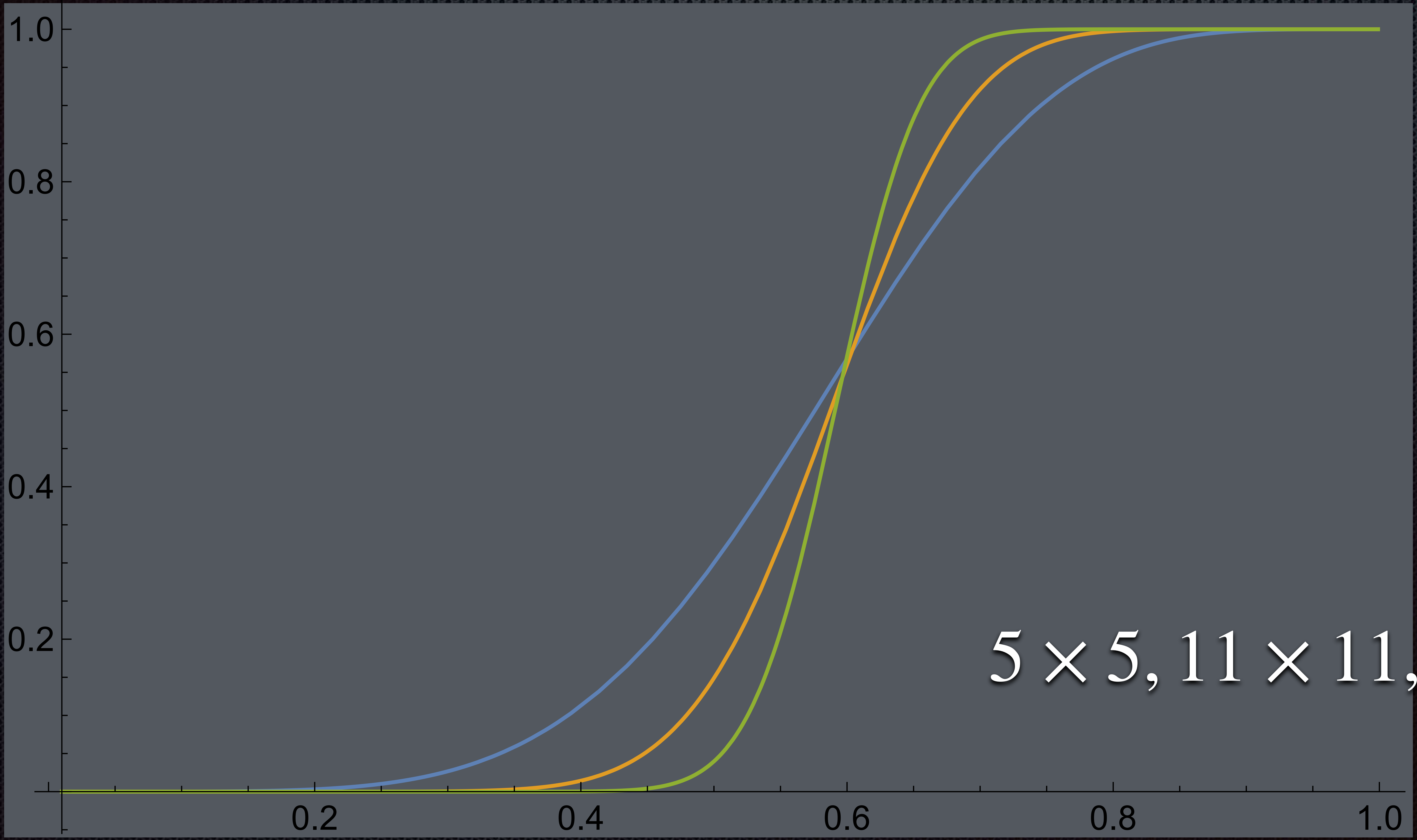

$5 \times 5, 11 \times 11, 22 \times 22$

$$R_{n,m}(z) = \sum_{k=0}^{nm} z^k A_{n,m}(k)$$

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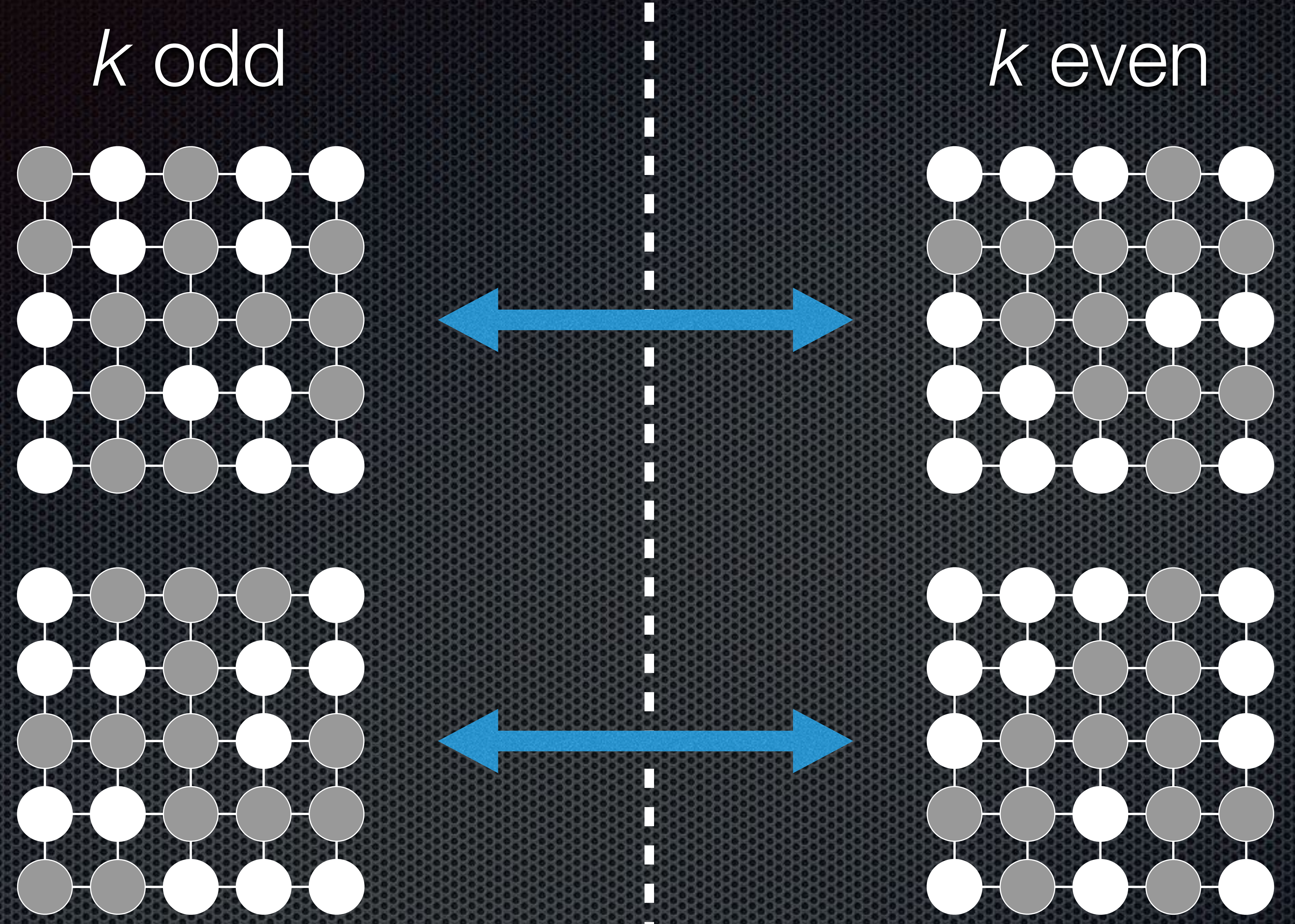
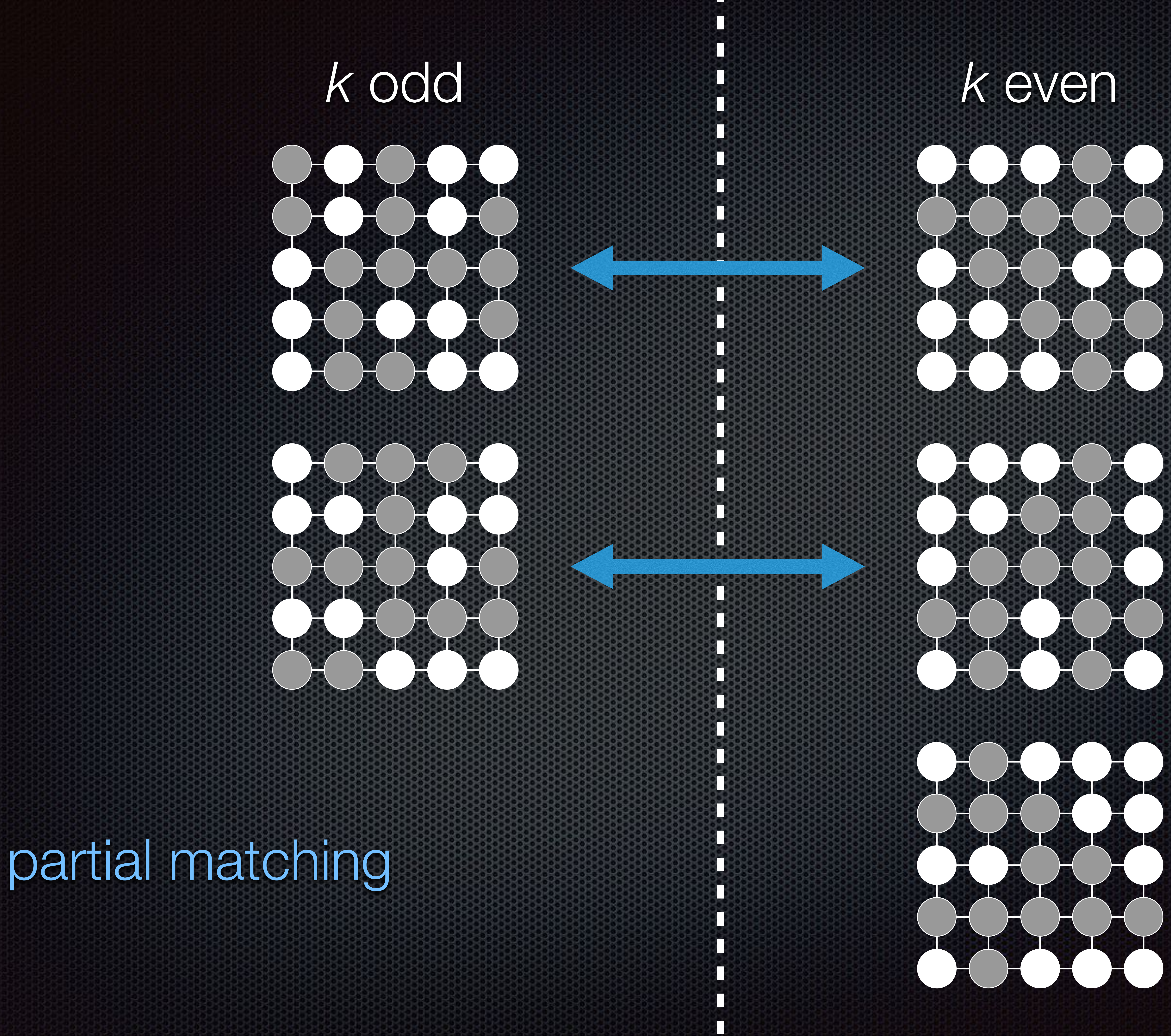
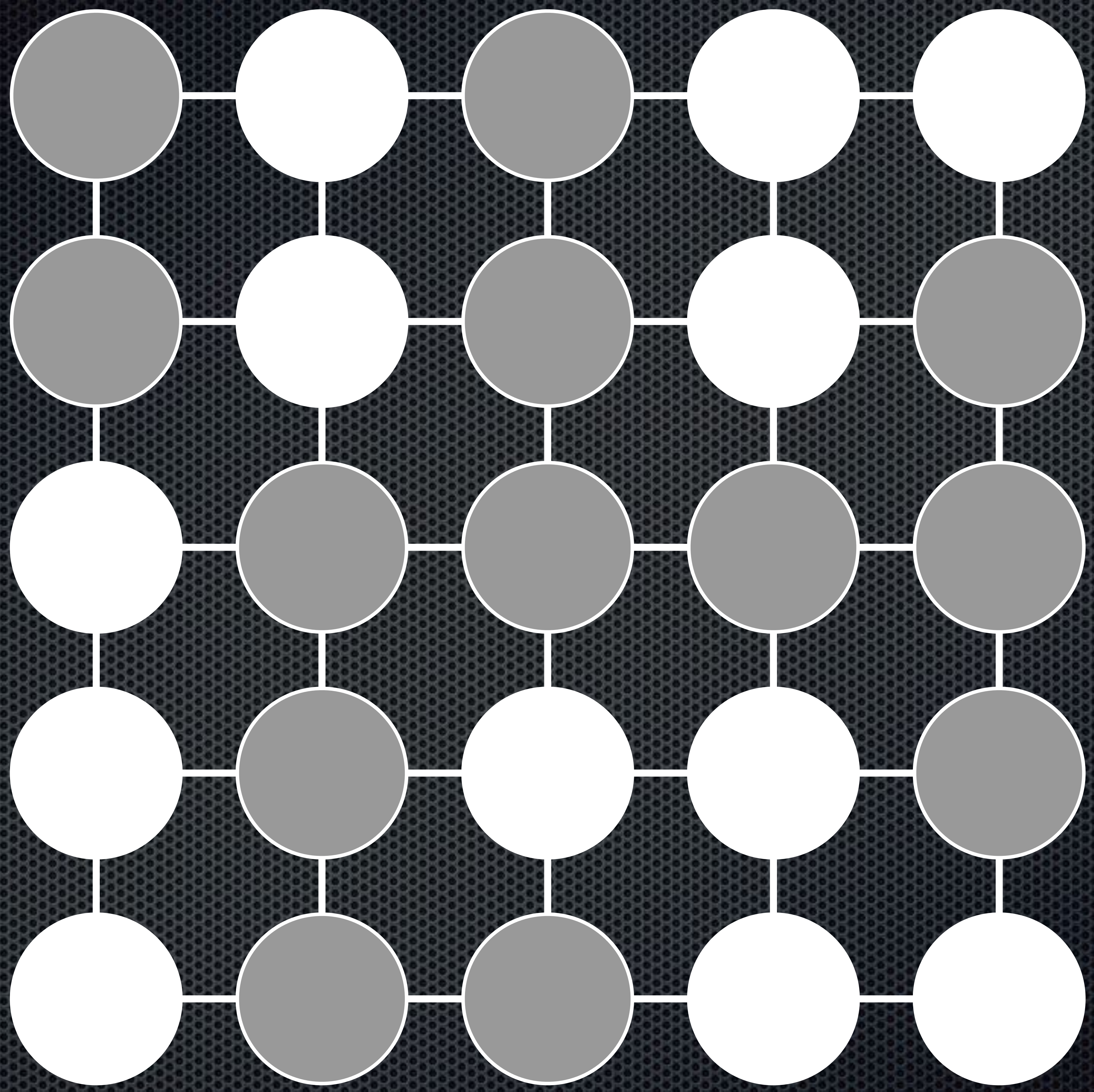
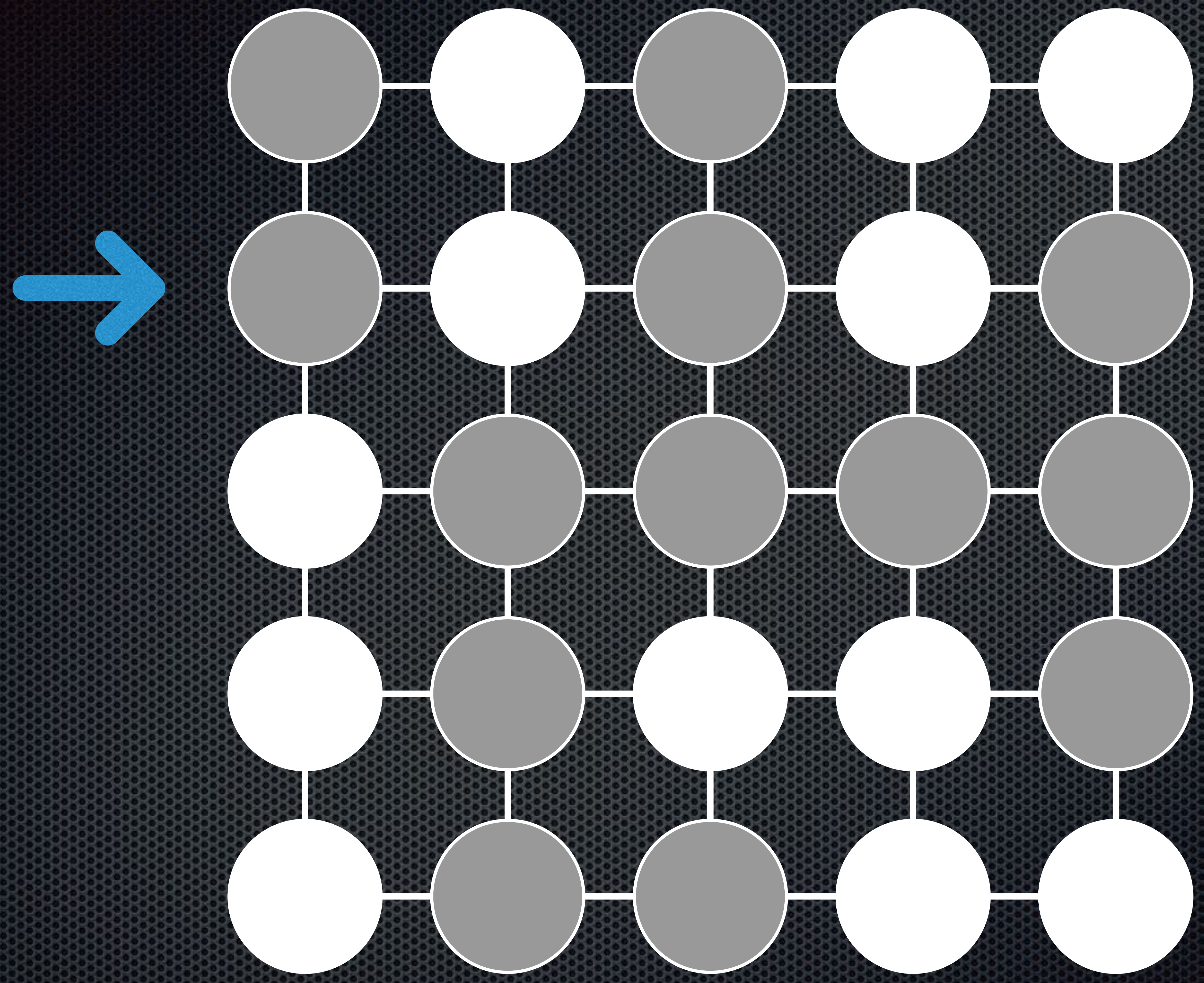
$m \backslash n$	1	2	3	4	5	6	7	8
1	−1	−1	−1	−1	−1	−1	−1	−1
2	1	−1	1	−1	1	−1	1	−1
3	−1	1	−1	1	−1	1	−1	1
4	1	1	1	1	1	1	1	1
5	−1	−1	−1	−1	−1	−1	−1	−1
6	1	−1	1	−1	1	−1	1	−1
7	−1	1	−1	1	−1	1	−1	1
8	1	1	1	1	1	1	1	1

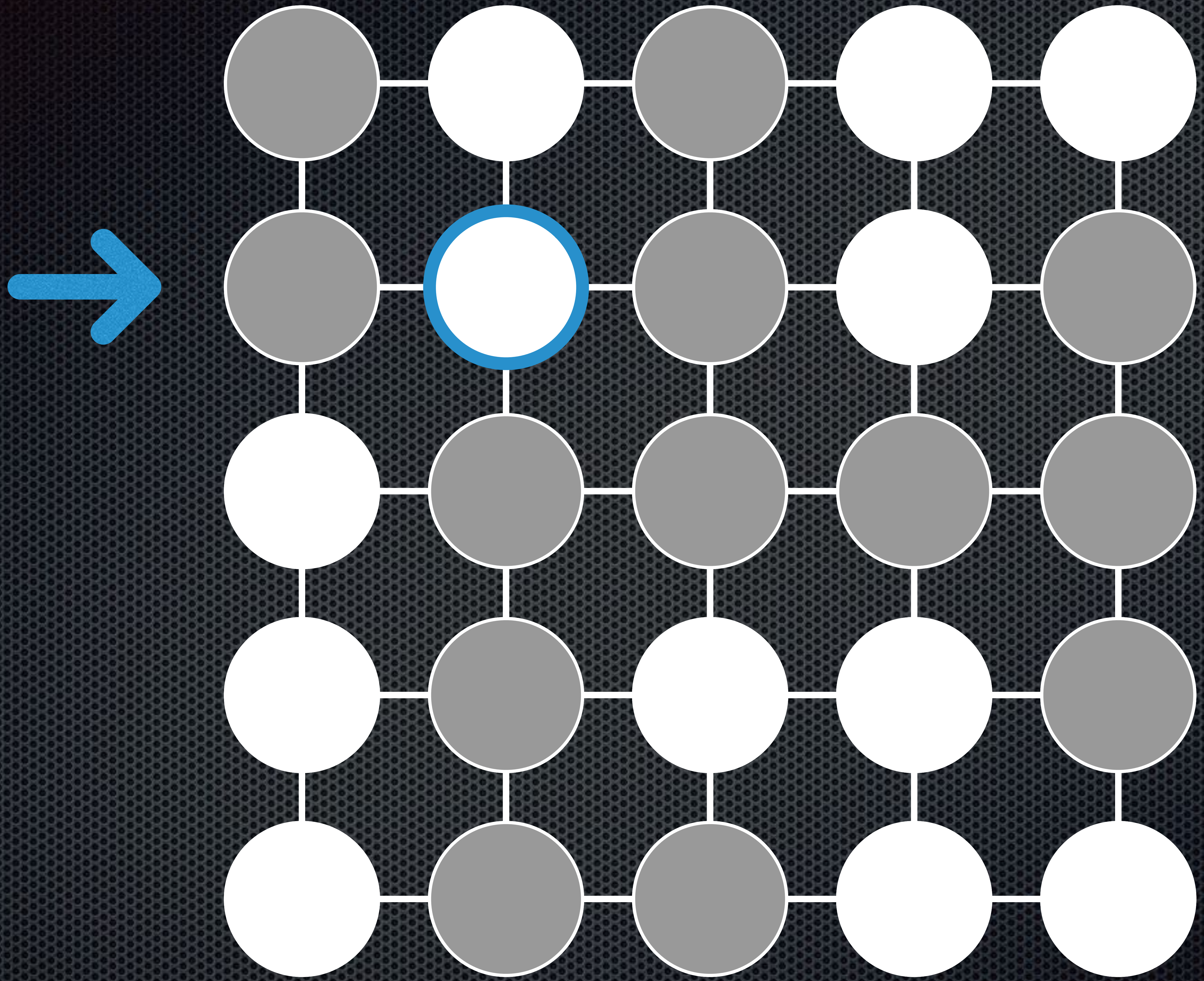


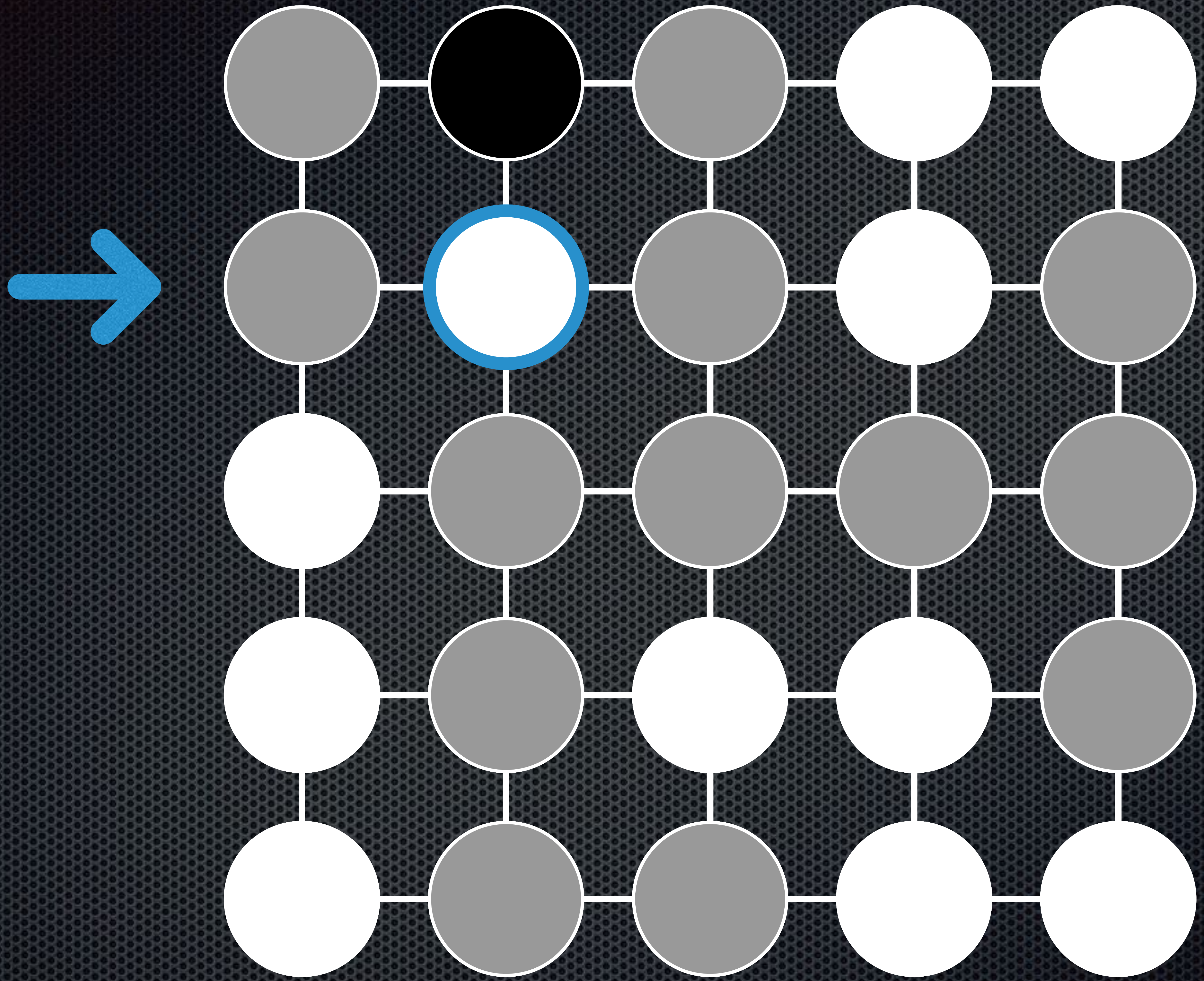
partial matching

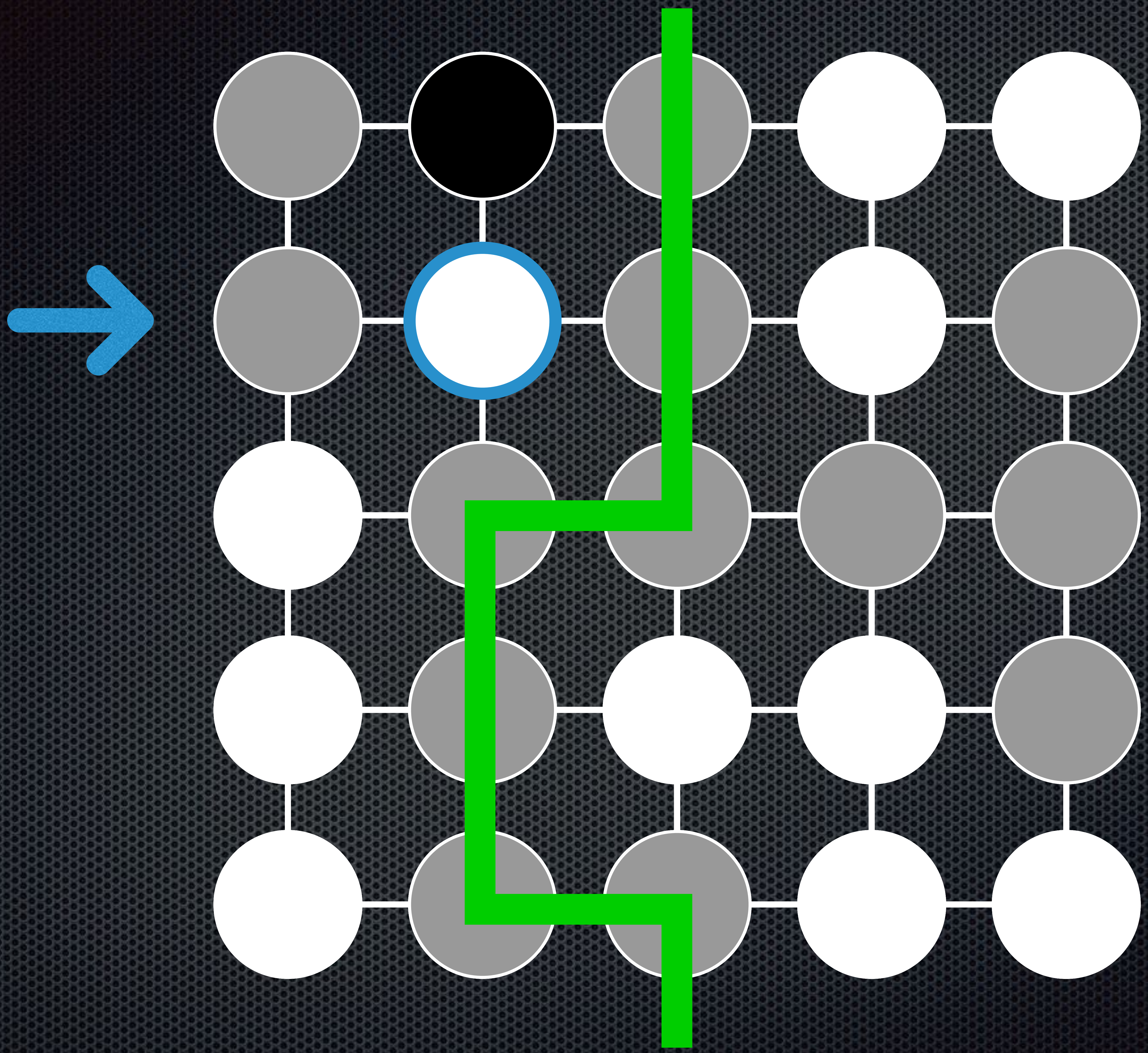

k odd
k even
partial matching

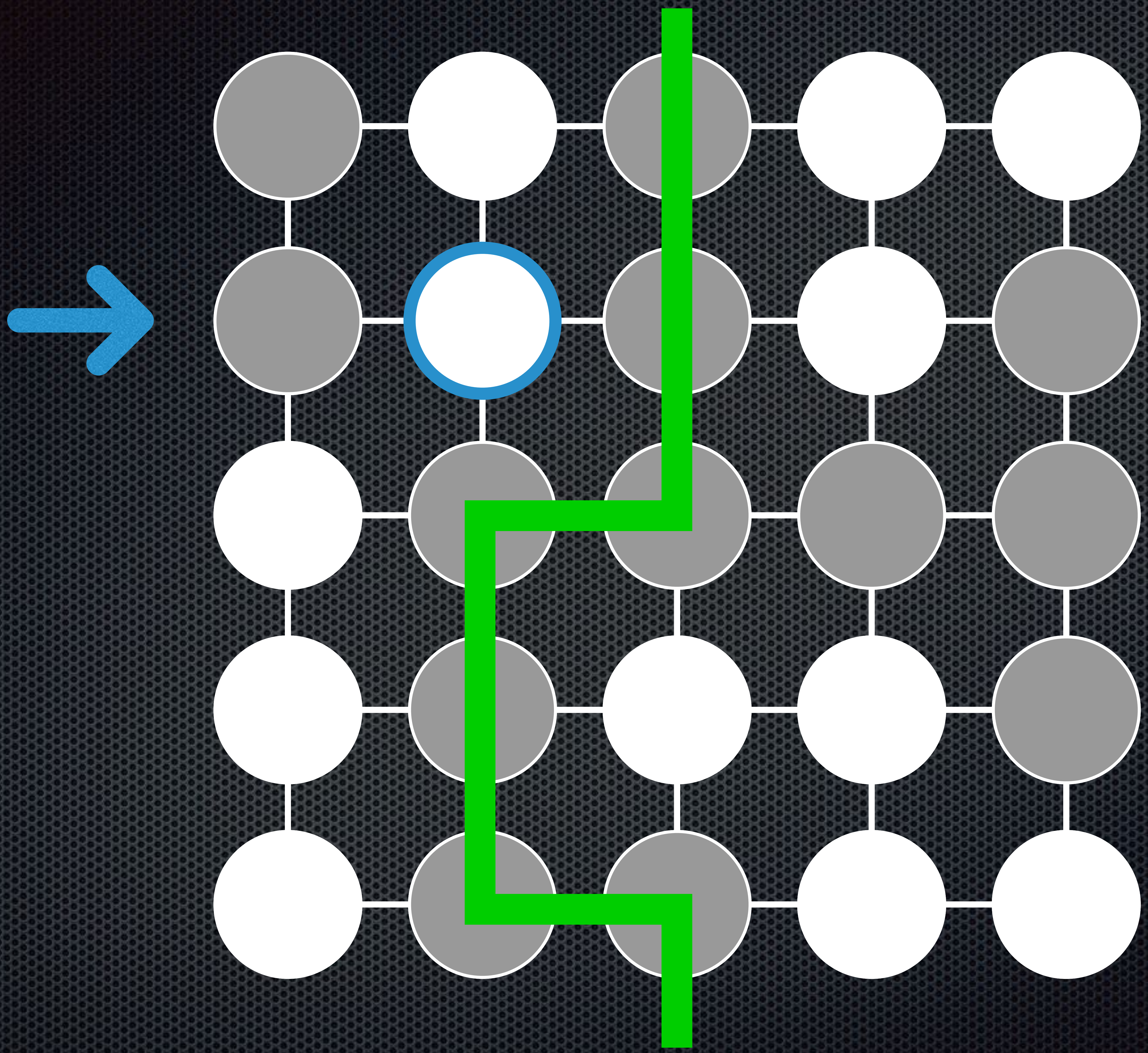


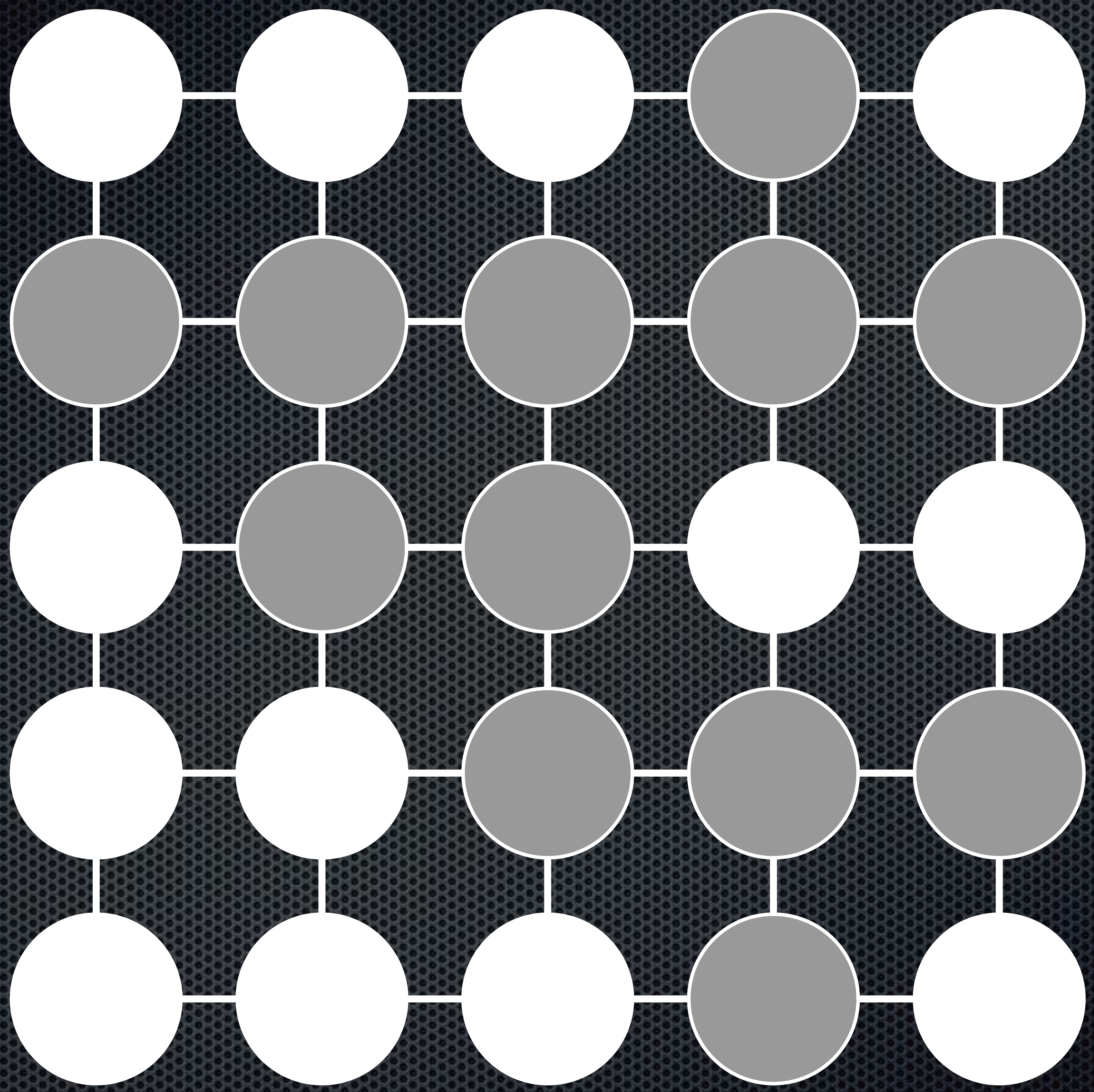


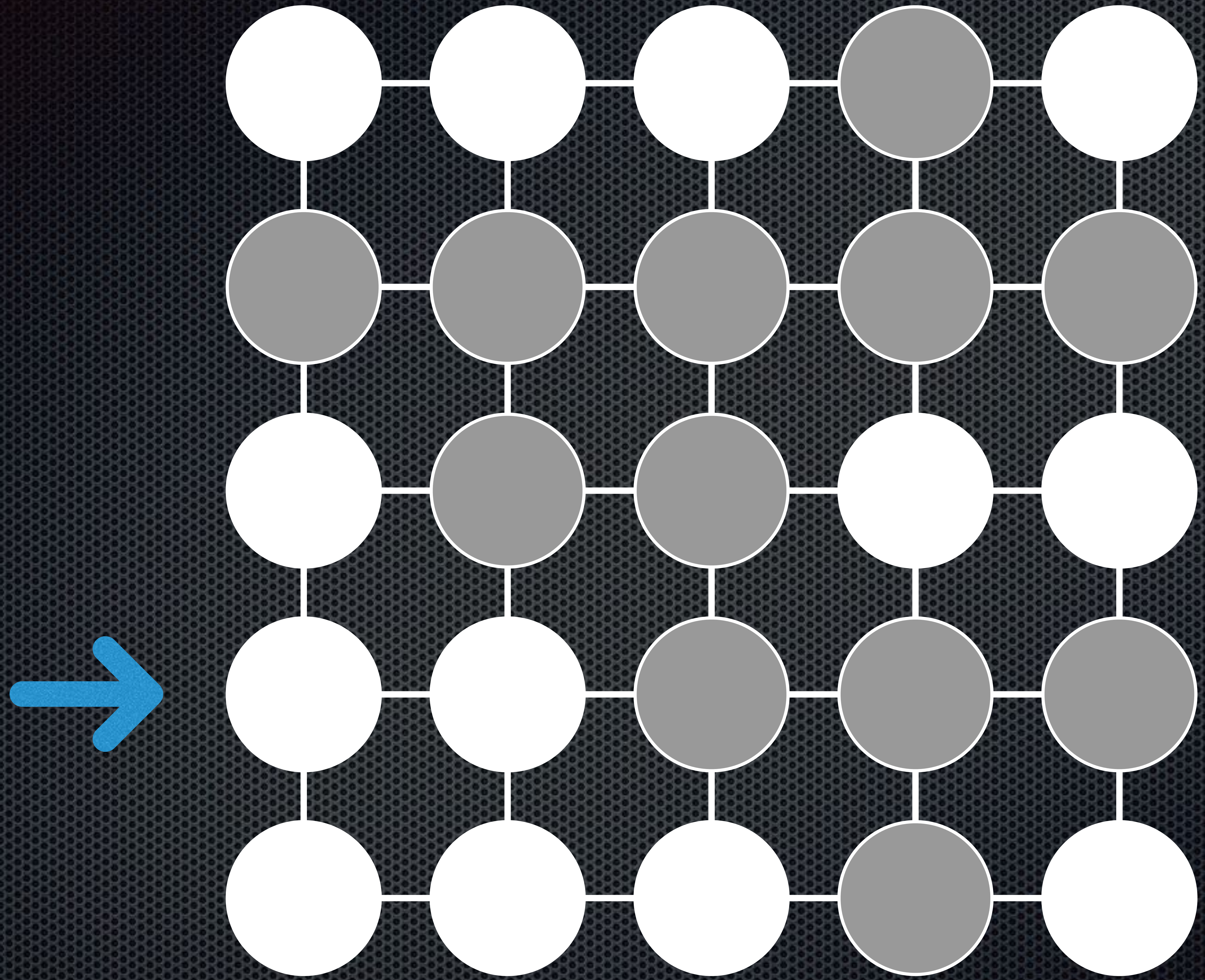


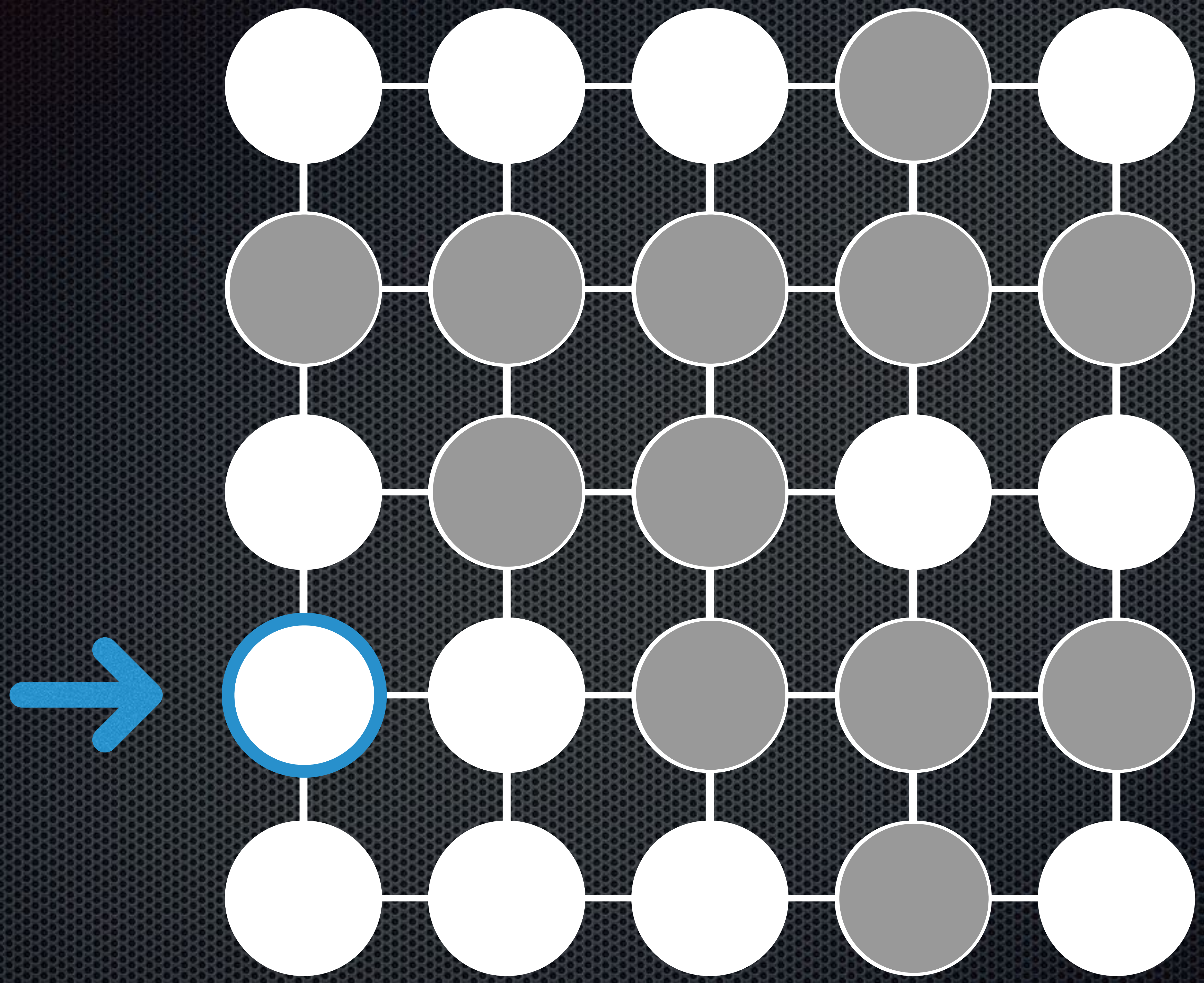


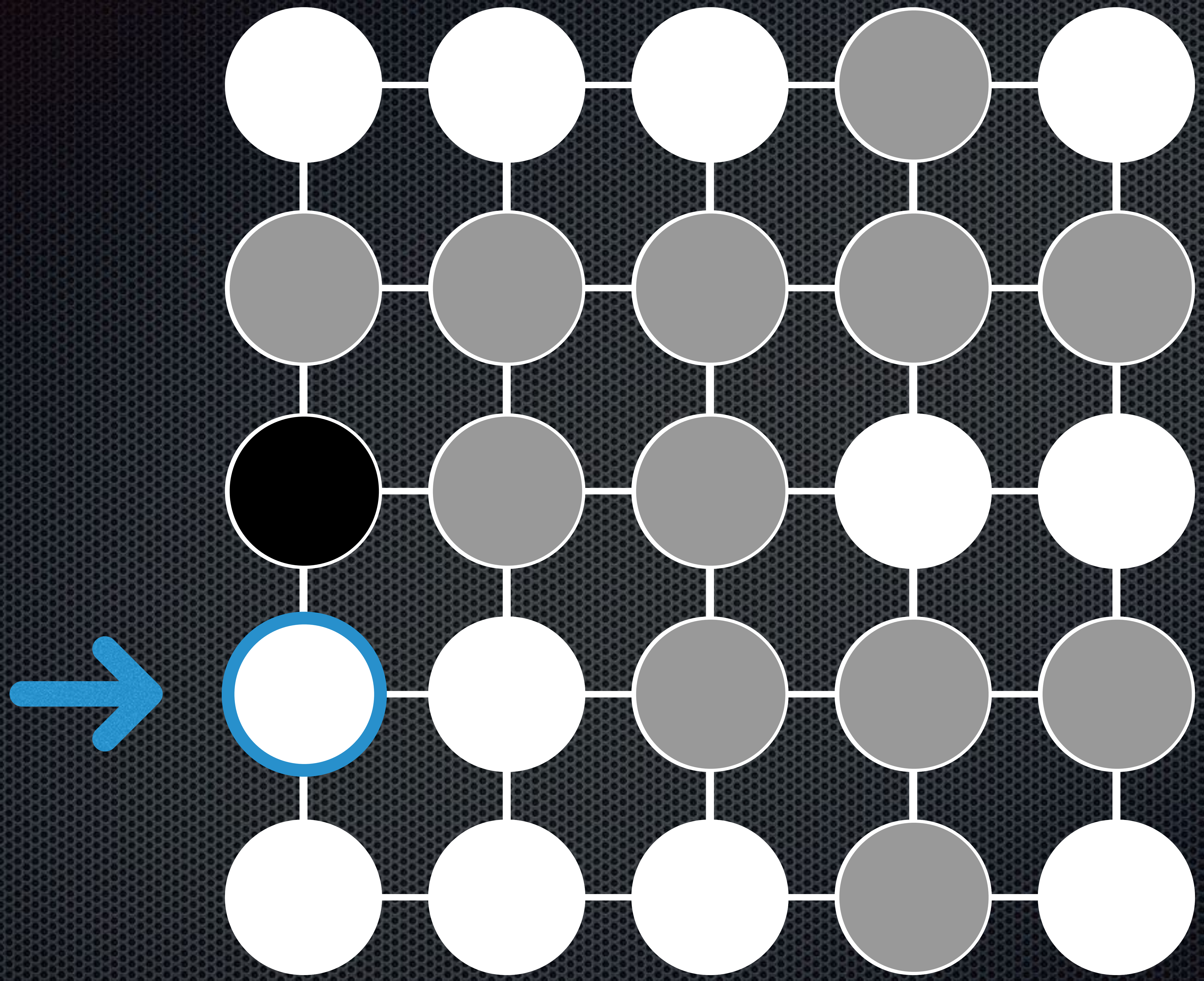


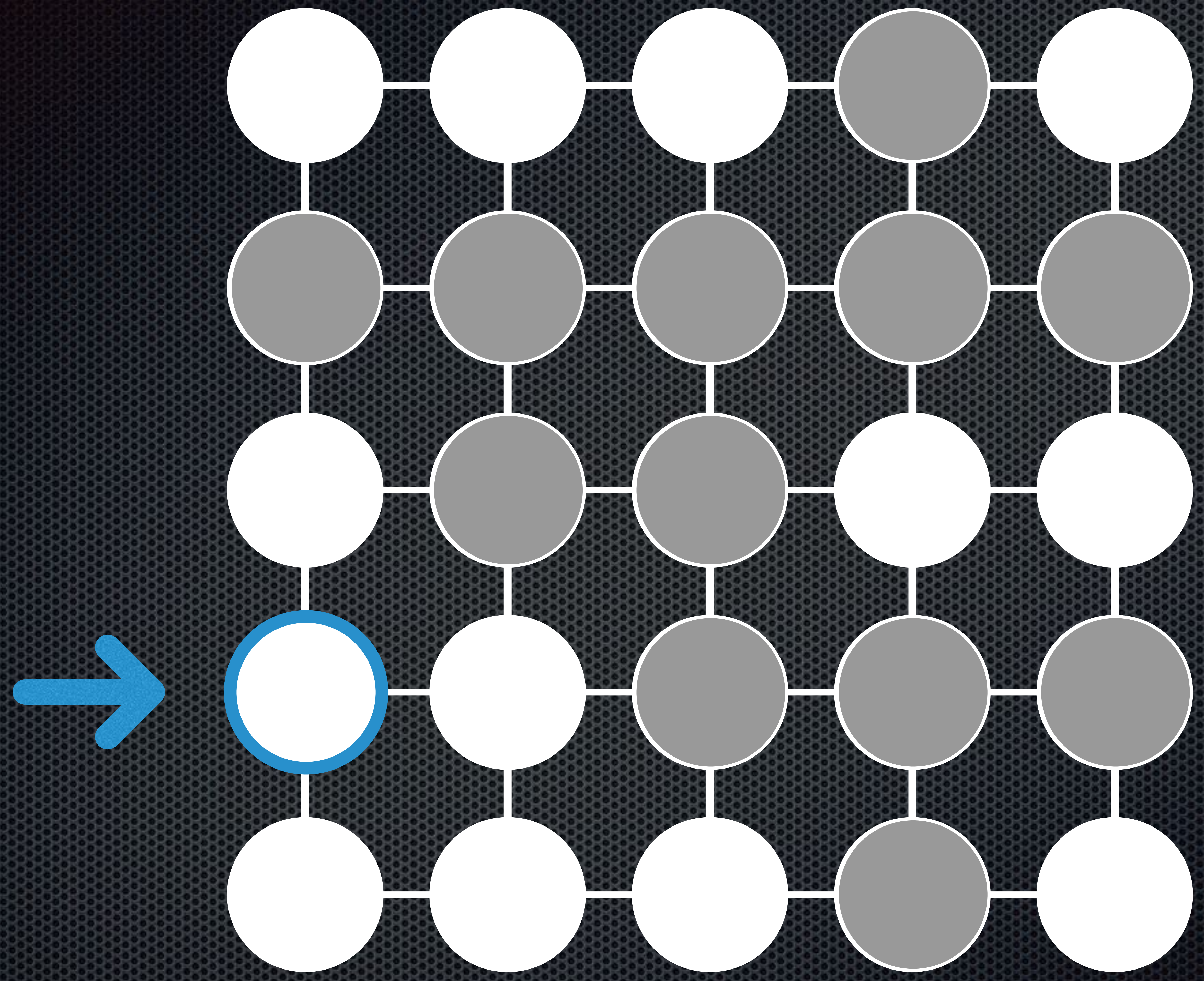


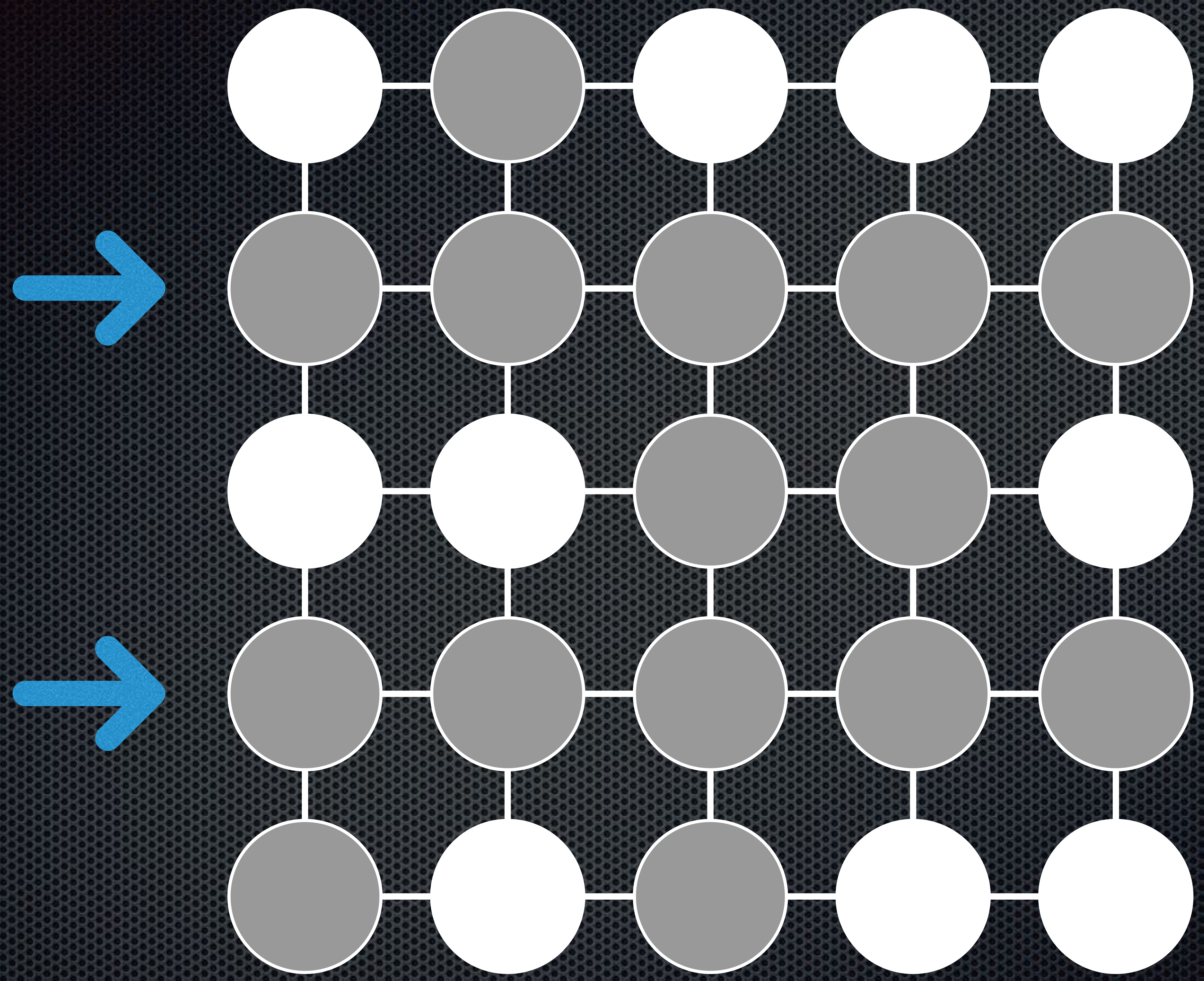


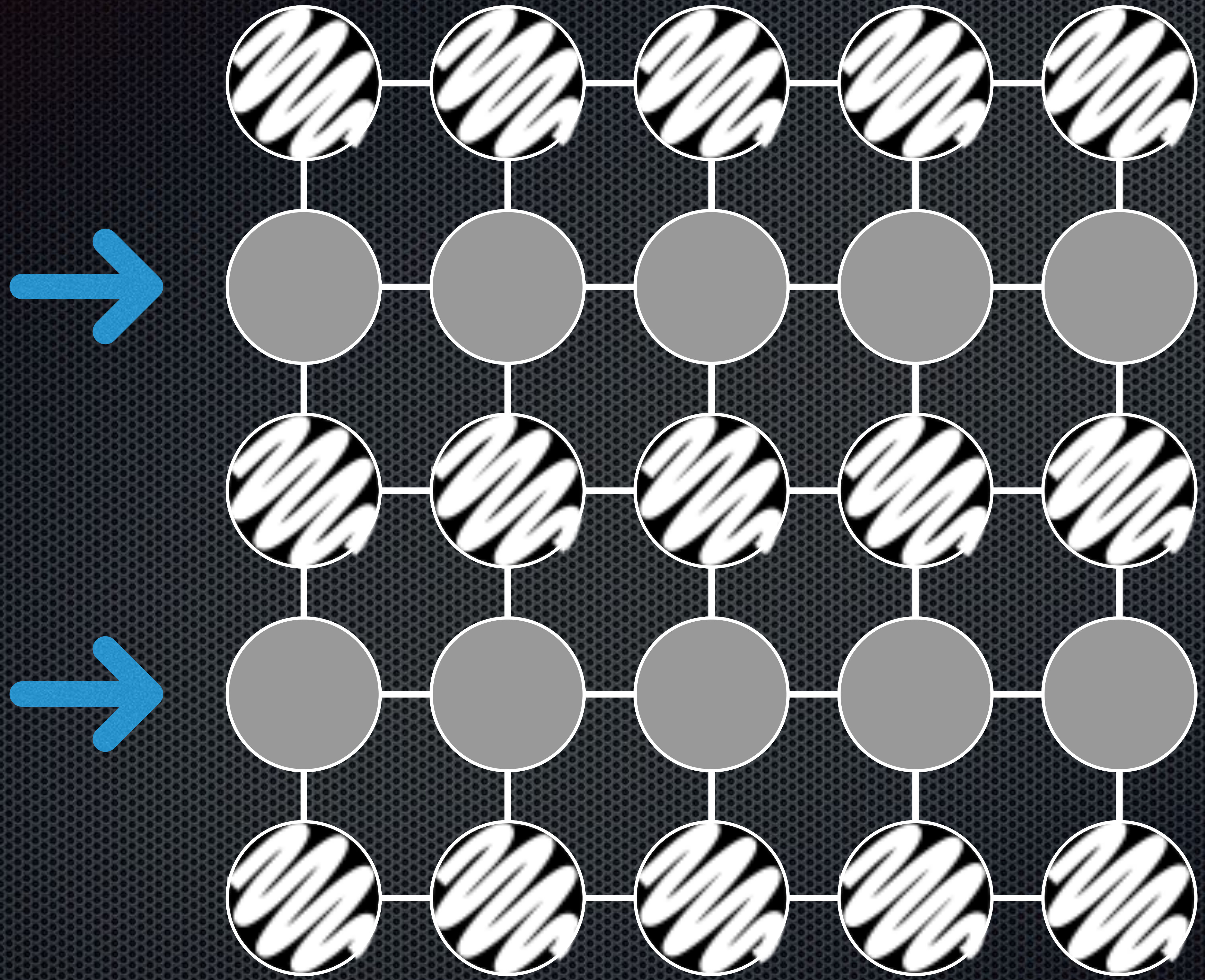


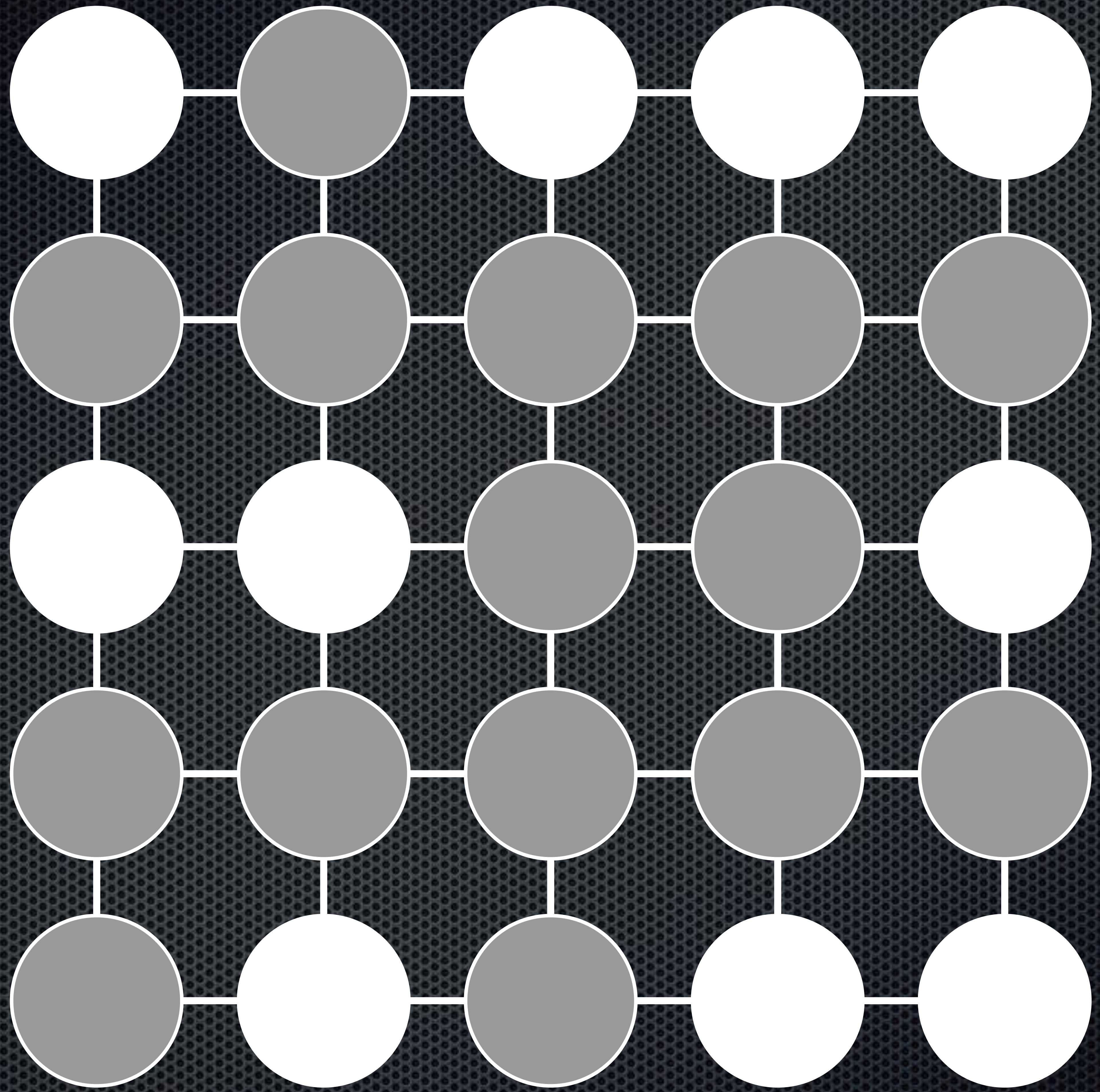


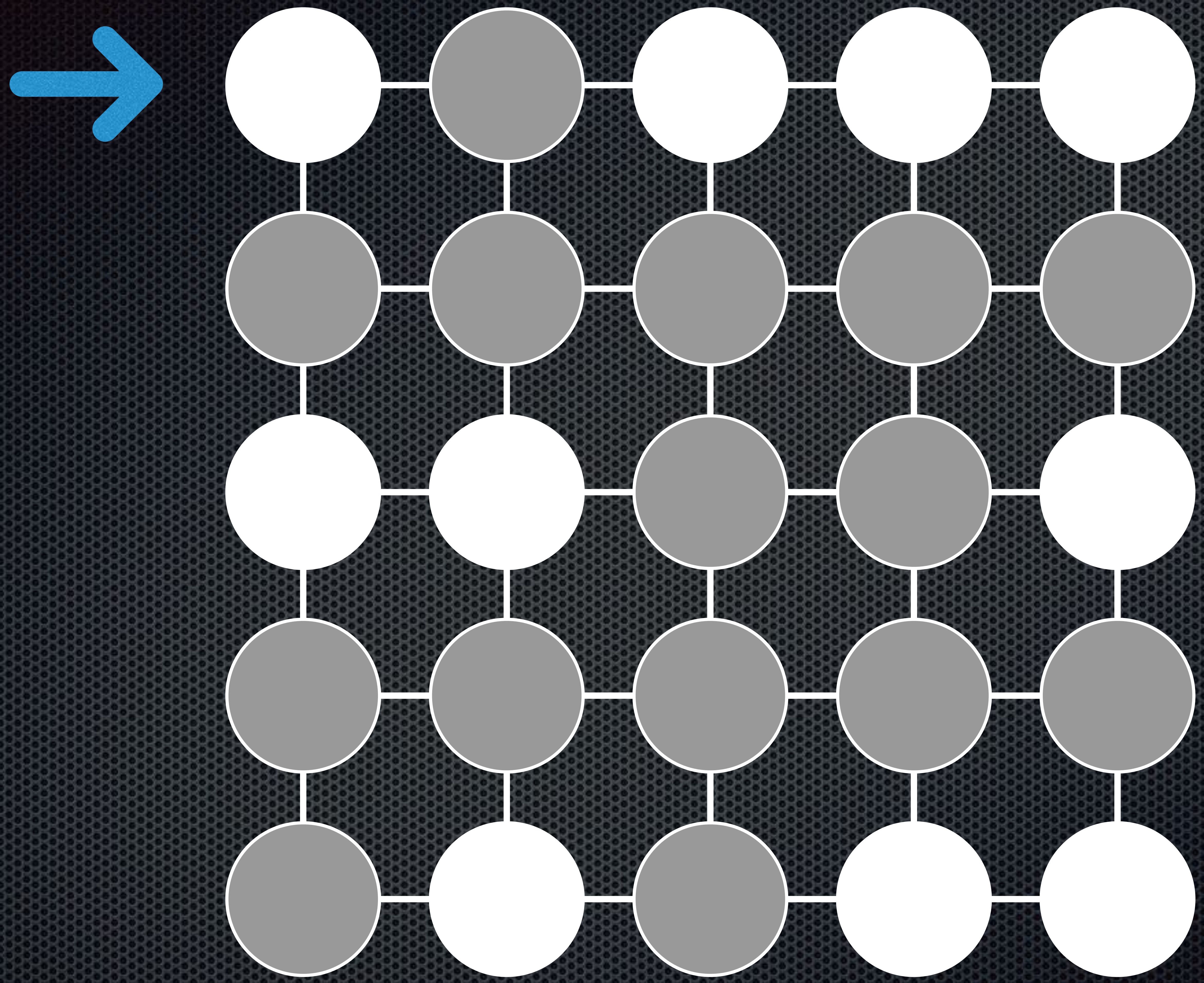


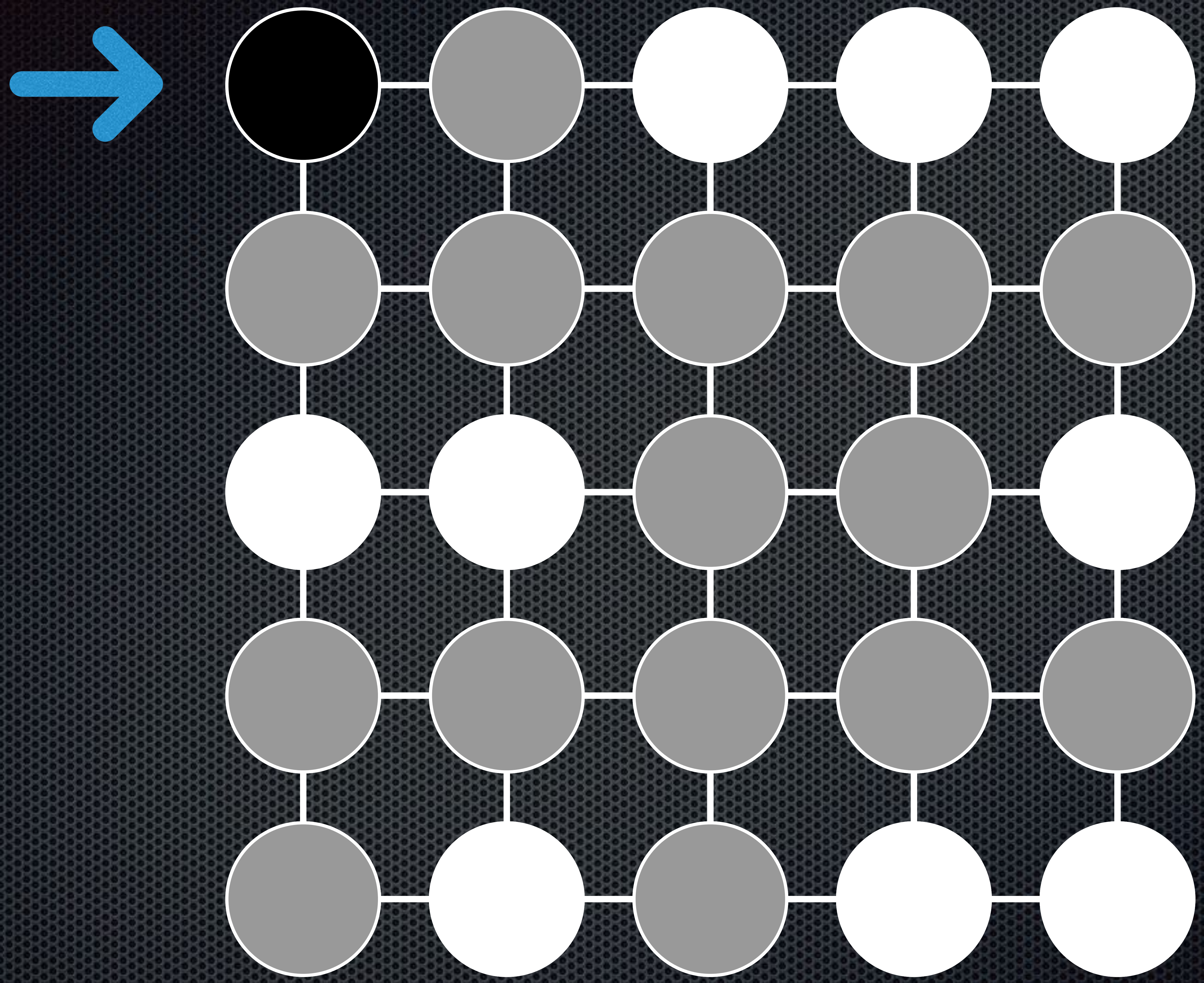


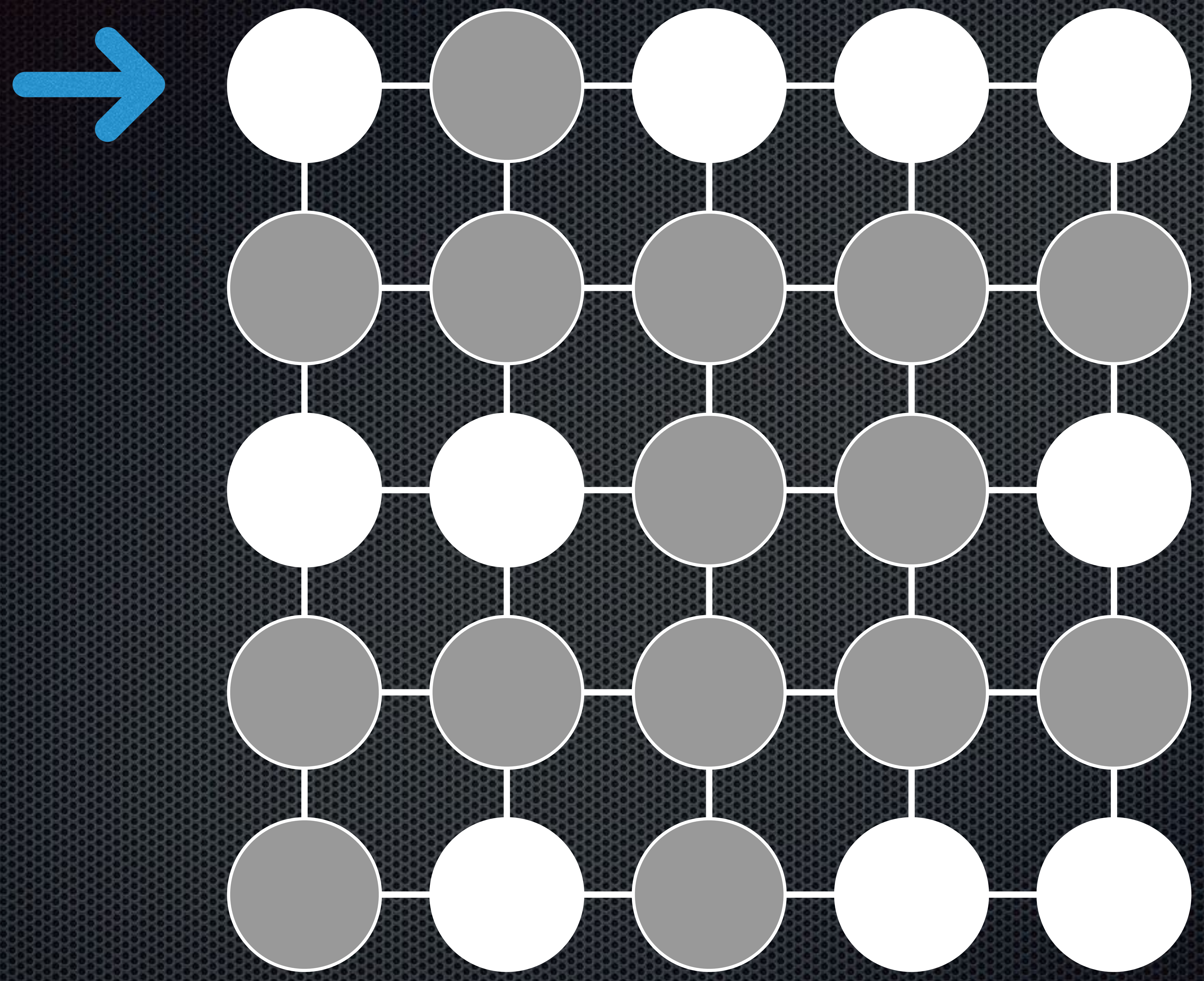


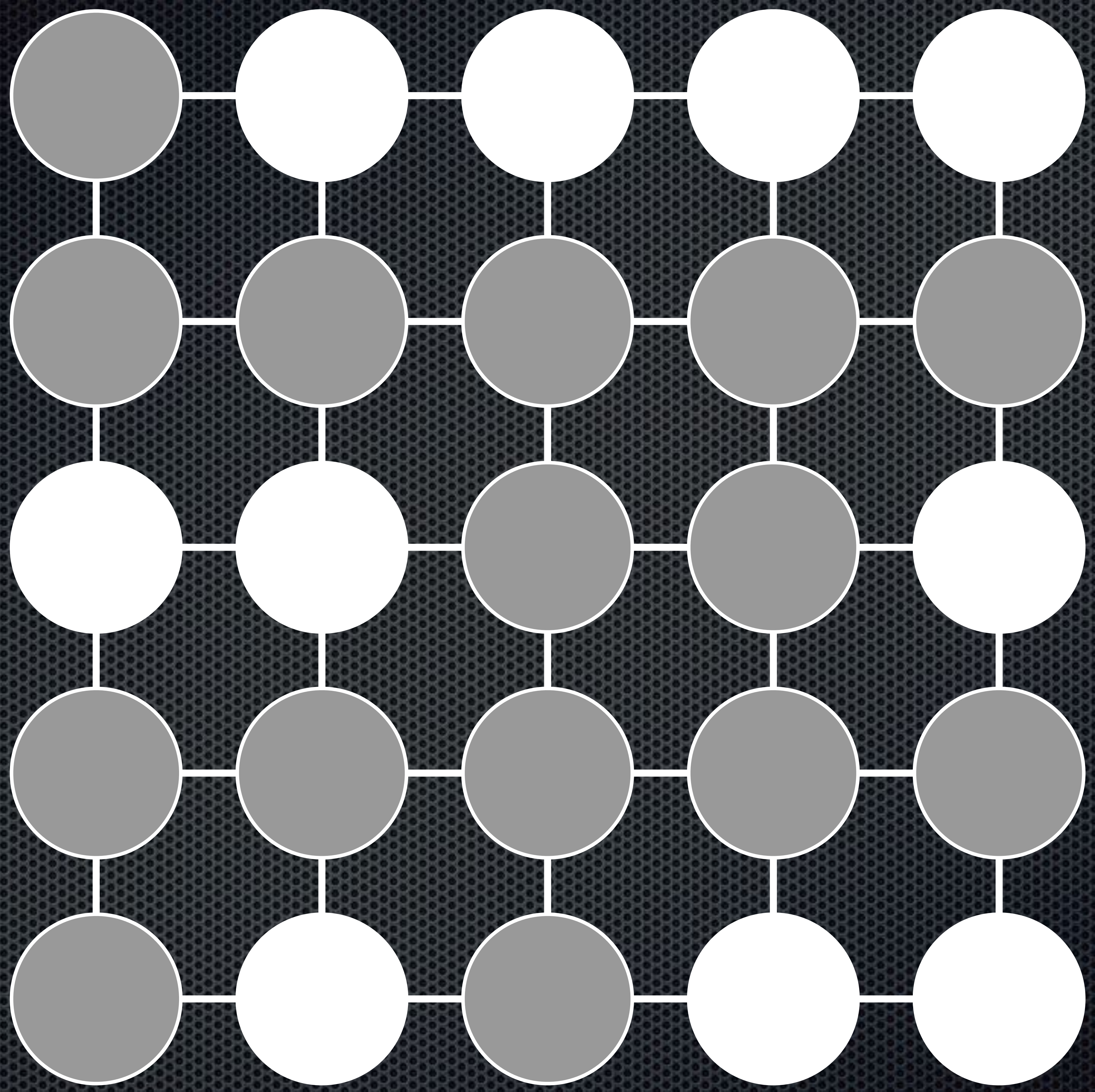


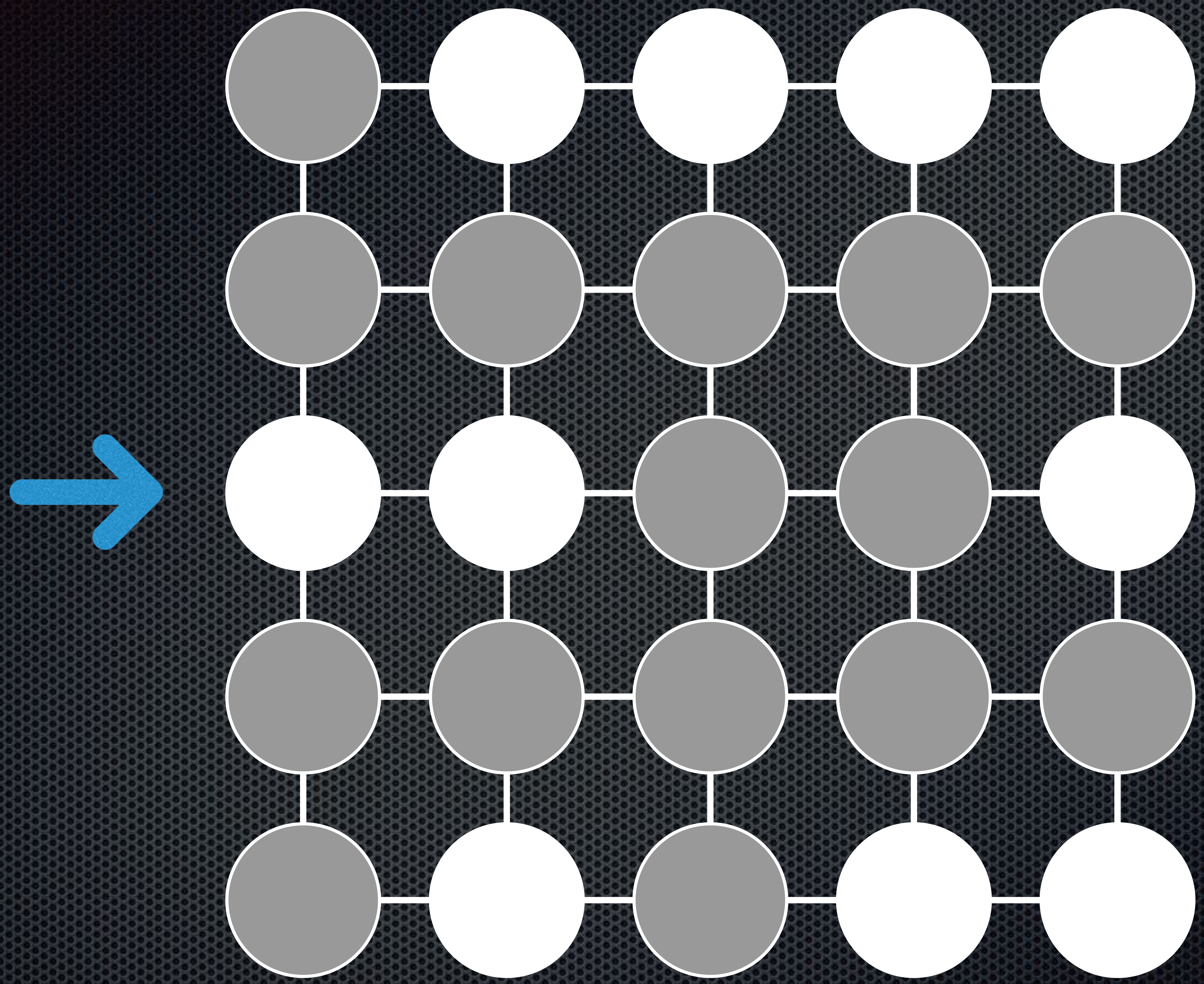


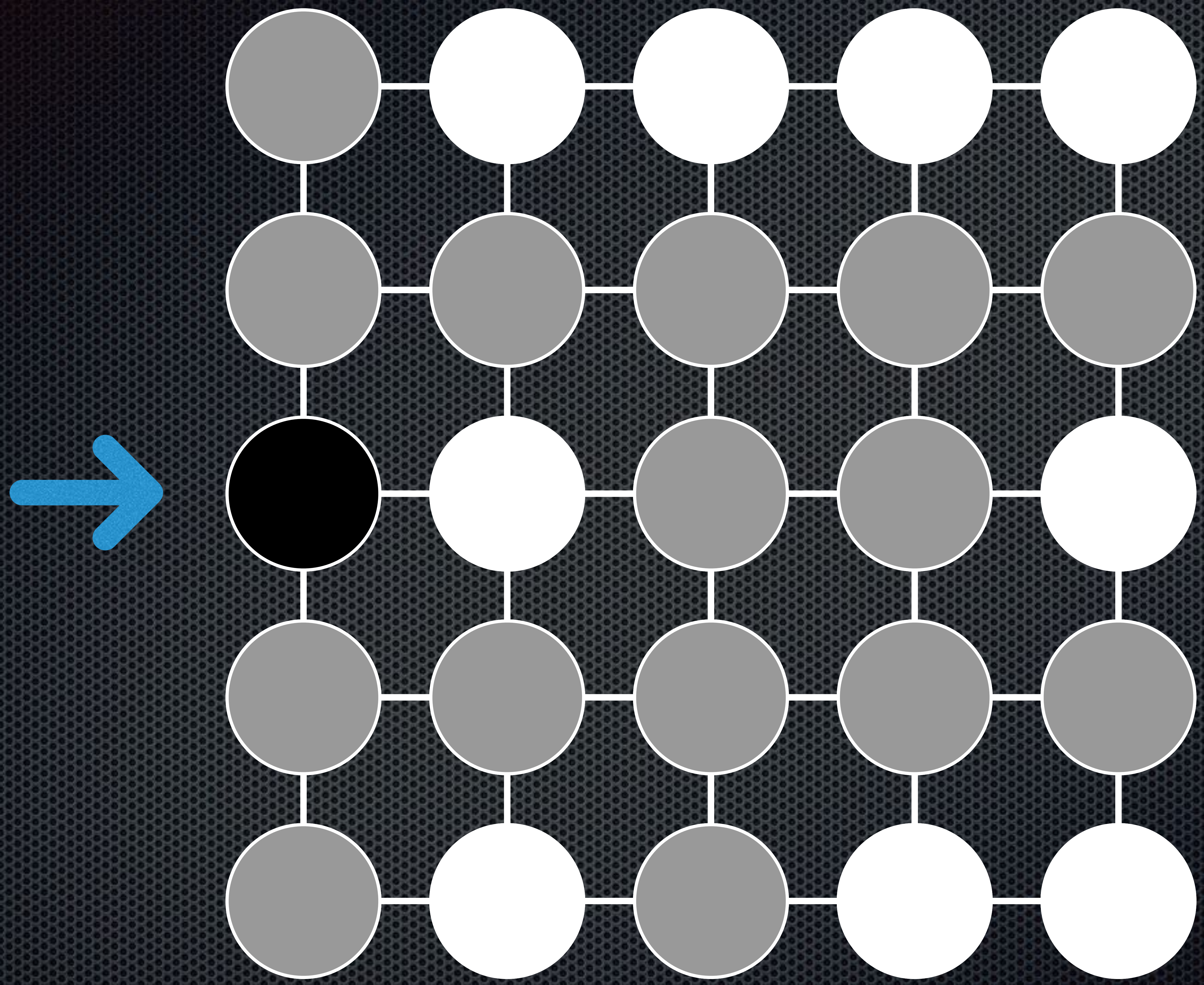


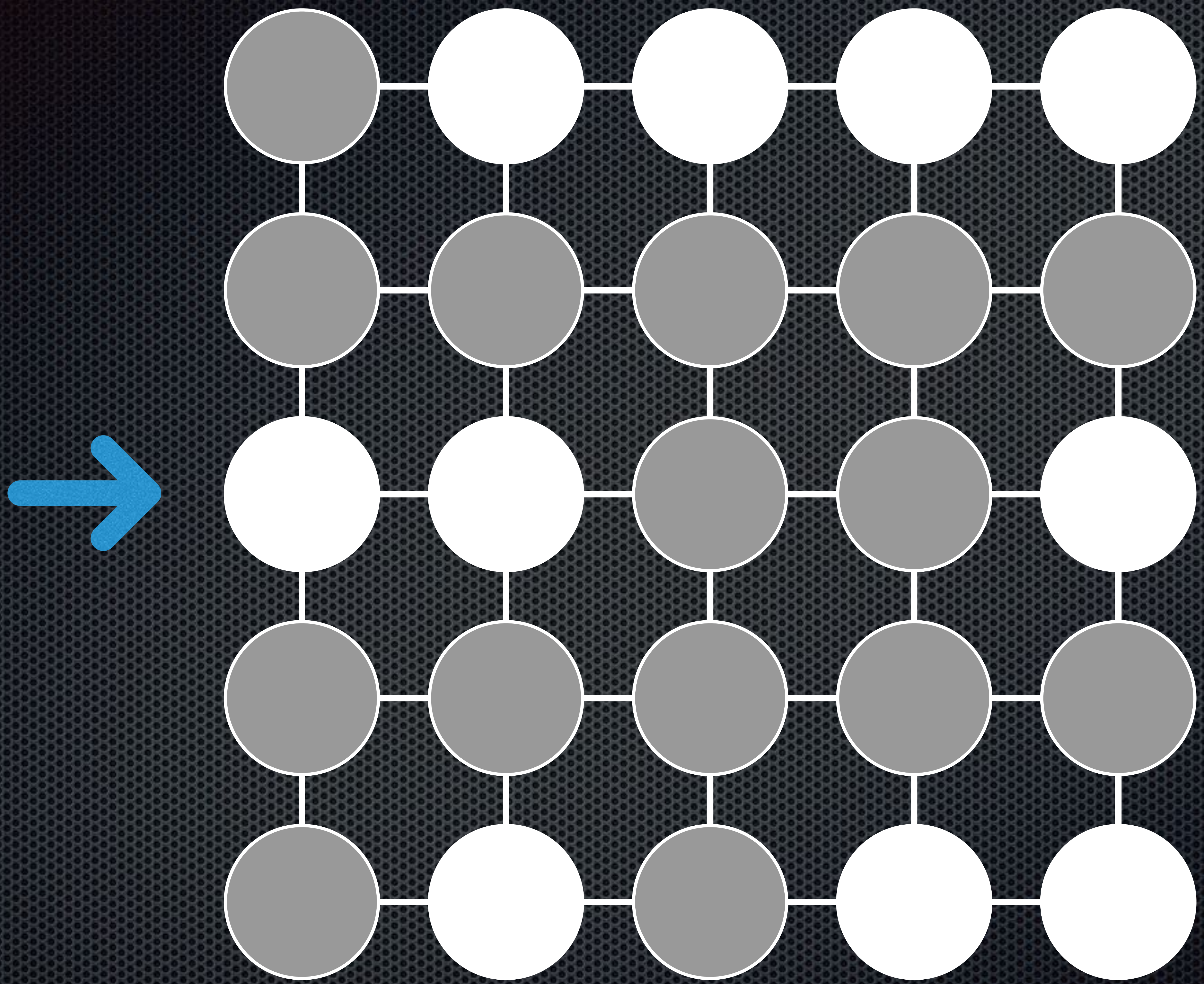




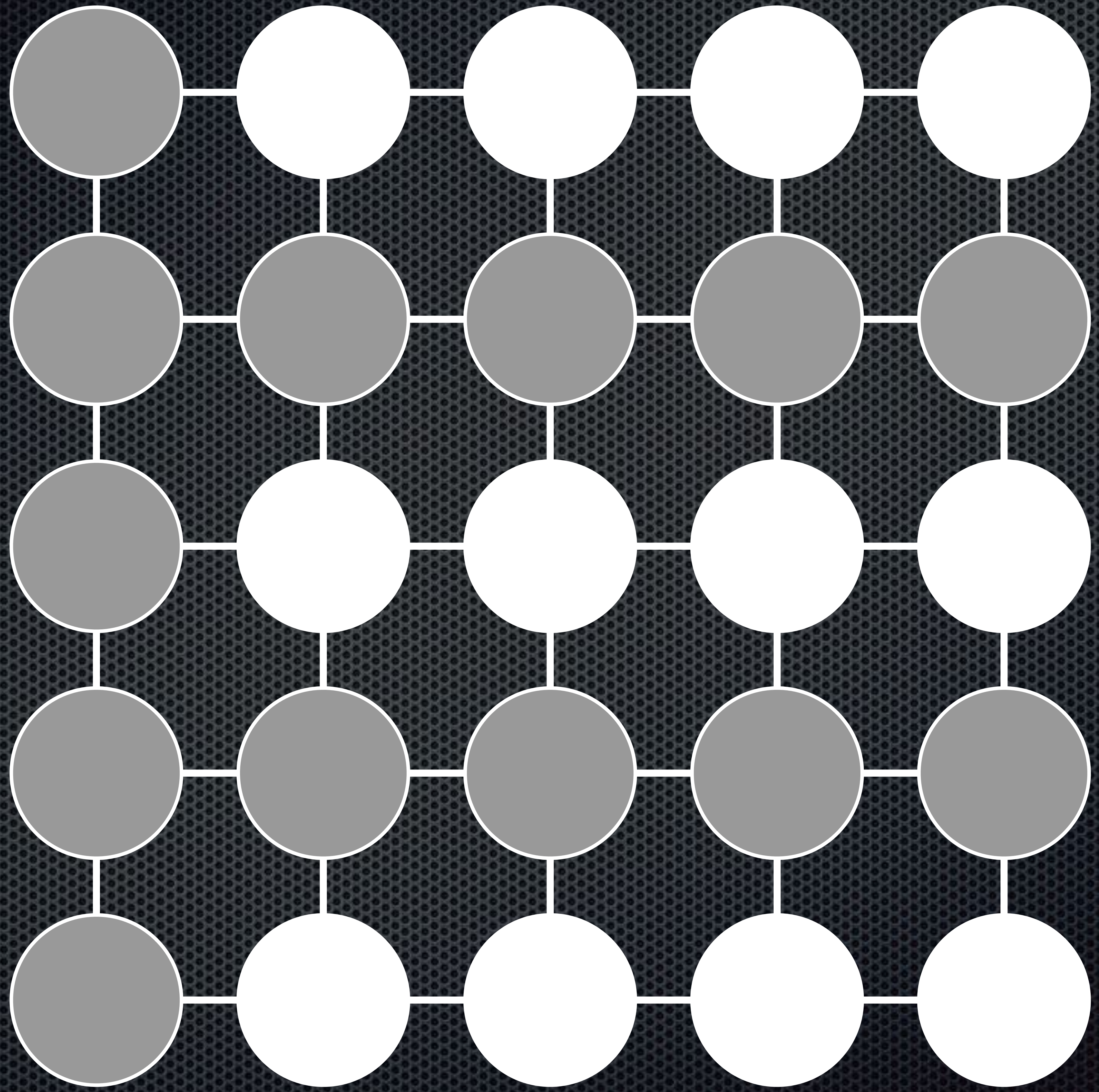






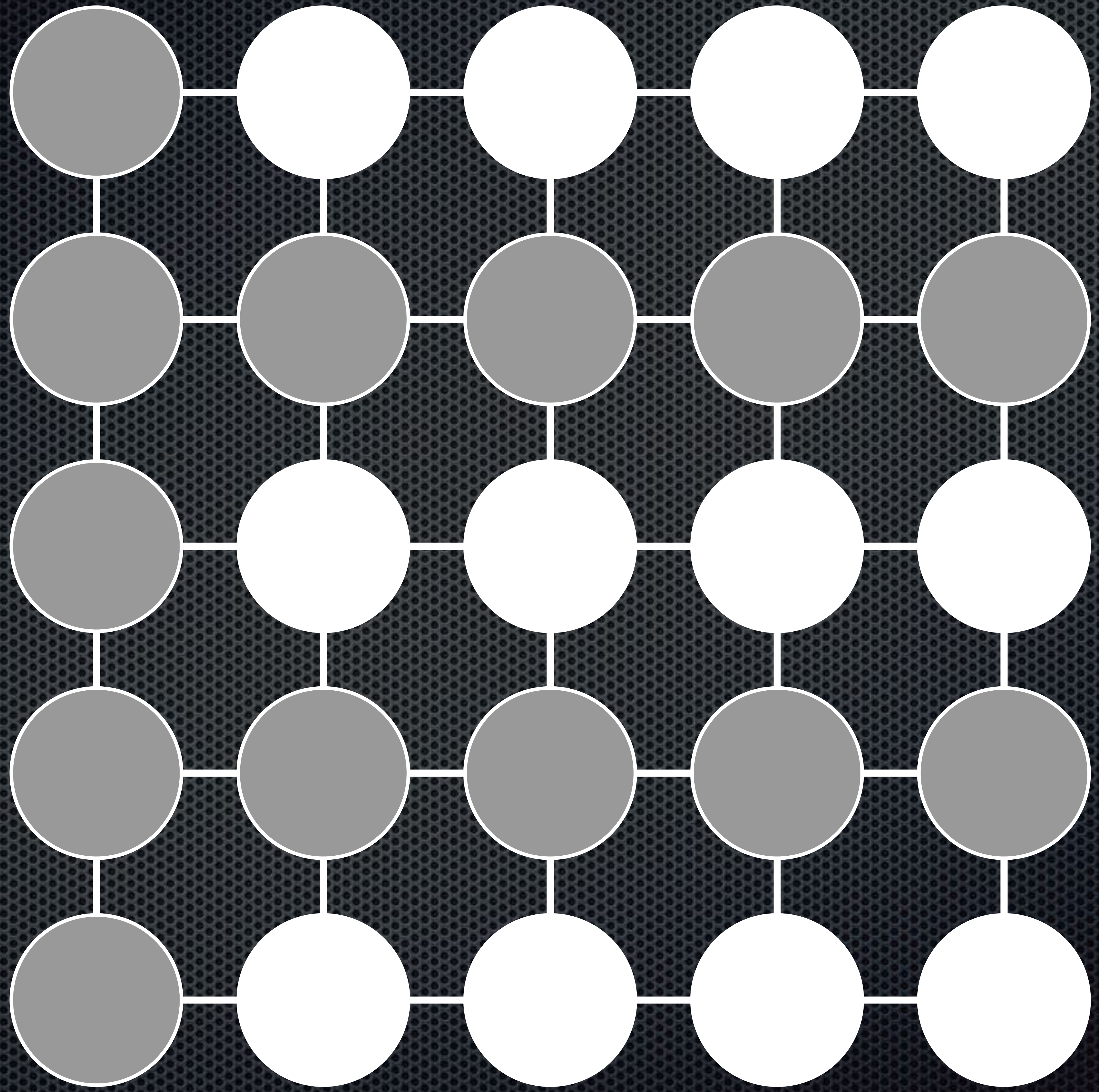


The Odd One Out



The Odd One Out

$$\left\lfloor \frac{m}{2} \right\rfloor n + \left\lceil \frac{m}{2} \right\rceil$$



$$R_{n,m}(-1) = \sum_{k \text{ even}} A_{n,m}(k) - \sum_{k \text{ odd}} A_{n,m}(k) = (-1)^{\lfloor \frac{m}{2} \rfloor n + \lceil \frac{m}{2} \rceil}$$

$m \backslash n$	1	2	3	4	5	6	7	8
1	−1	−1	−1	−1	−1	−1	−1	−1
2	1	−1	1	−1	1	−1	1	−1
3	−1	1	−1	1	−1	1	−1	1
4	1	1	1	1	1	1	1	1
5	−1	−1	−1	−1	−1	−1	−1	−1
6	1	−1	1	−1	1	−1	1	−1
7	−1	1	−1	1	−1	1	−1	1
8	1	1	1	1	1	1	1	1

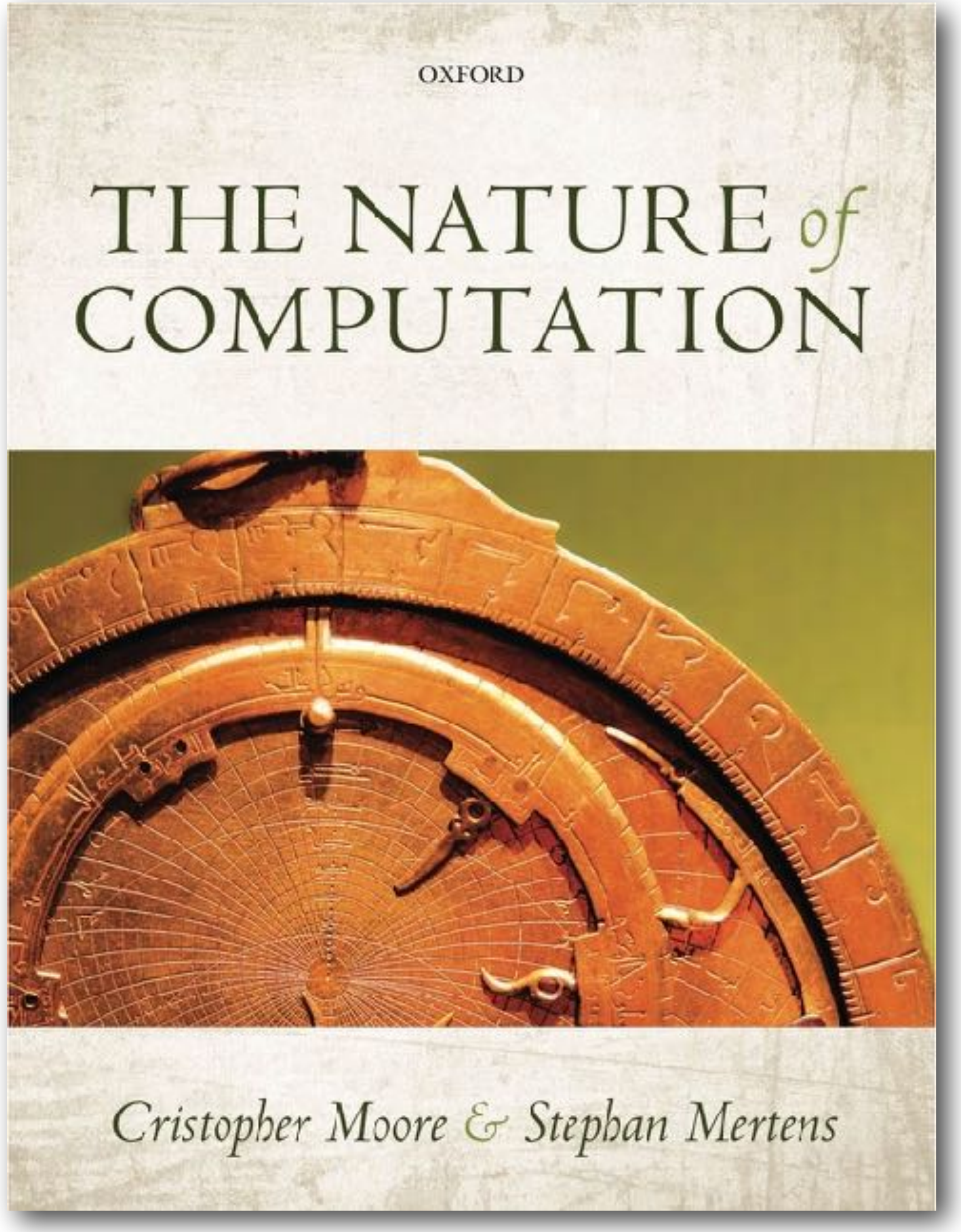
Other Matching Proofs

A square integer has an odd number of divisors

The number of binary trees with 2^n leaves is odd

A prime $p = 4n + 1$ has an odd number of representations $p = x^2 + y^2$

Shameless plug



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To put it bluntly: this book rocks! It somehow manages to combine the fun of a popular book with the intellectual heft of a textbook.

Scott Aaronson, UT Austin

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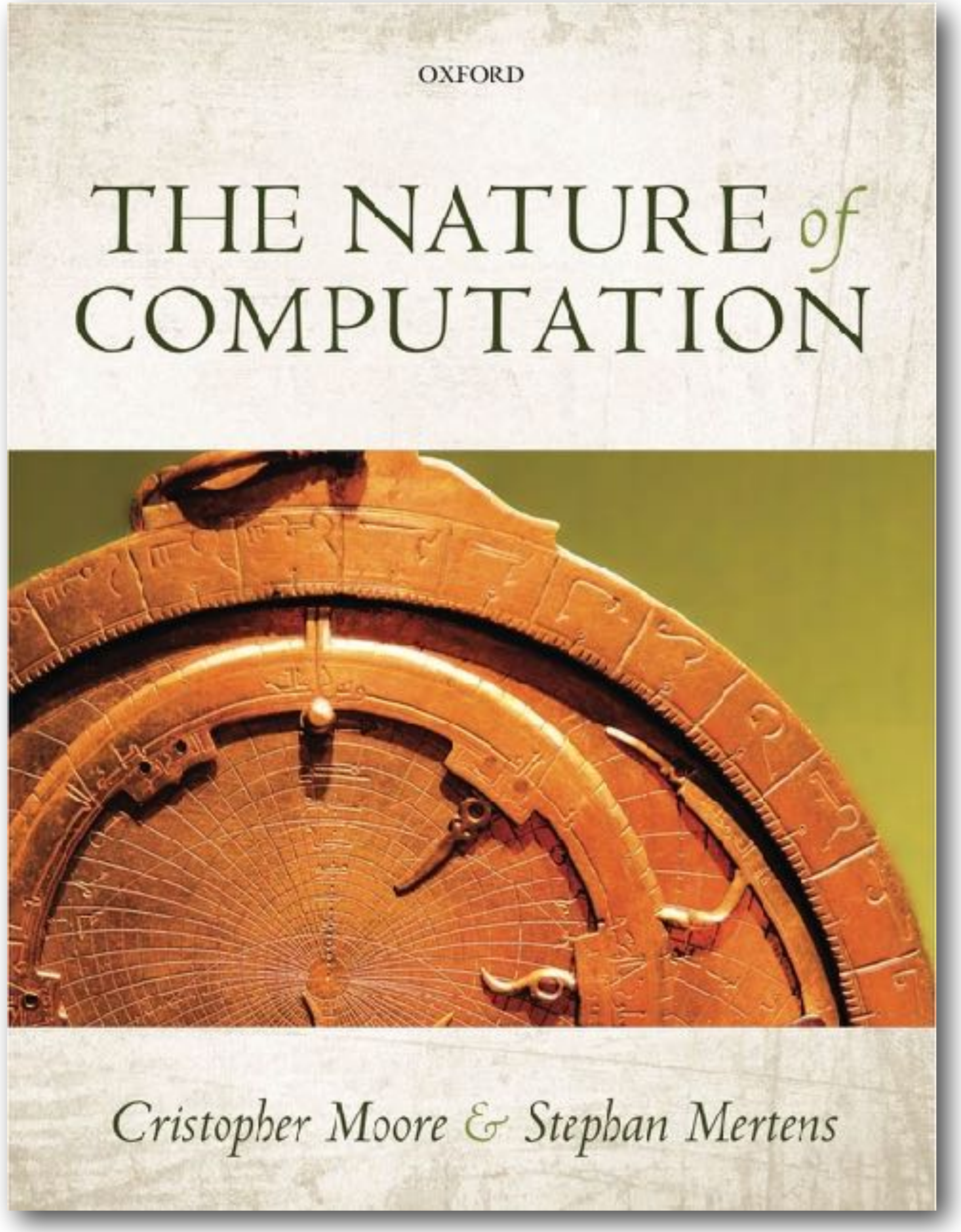

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