Removing partisan bias in redistricting: computational complexity meets the science of gerrymandering[+]

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Gerrymandering

Creation of district plans with *highly asymmetric* electoral outcomes to *disenfranchise* voters

- **Long history starting from as early as 1812**
  1812 : shape of South Essex district (Massachusetts) resembling a ***salamander*** created to favor selected candidates

- **Extensive legal history too!**

  1986: US Supreme Court : gerrymandering *is justiciable*
  2006*:* US Supreme Court : *some measure* of partisan symmetry *may be used* to remedy gerrymandering
  Which measure? Court did not say. Depends case by case.
  2019: US Supreme Court : best settled at the legislative and political level (ALAS!)

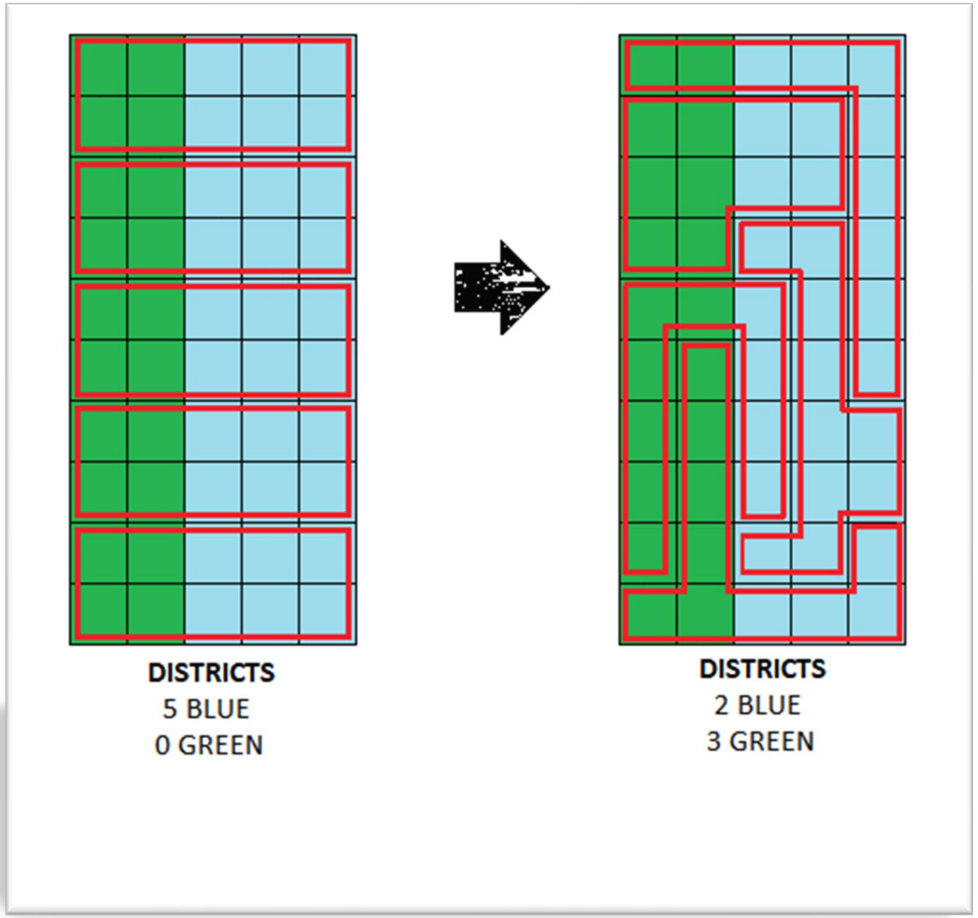
- **Major impediment to removing gerrymandering**
  How to formulate an effective and precise measure for partisan bias that will be **acceptable in courts** ?

"Gerry" and "salamander" 1812, State Senate Elections, Massachusetts

Some tools politicians use for partisan gerrymandering in 2-party system

- ❑ **Packing** → concentrate voters of opposition party in a single district

- ❑ **Cracking** → spread voters of opposition party across many districts



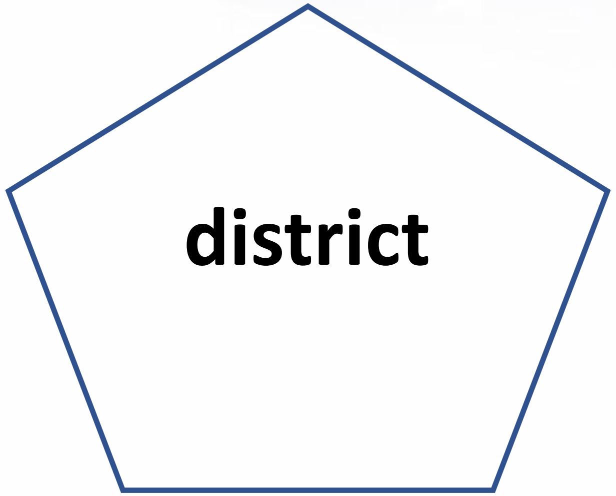
Other methods include

- Hijacking
- Kidnapping etc.

"Efficiency Gap" measure for partisan gerrymandering

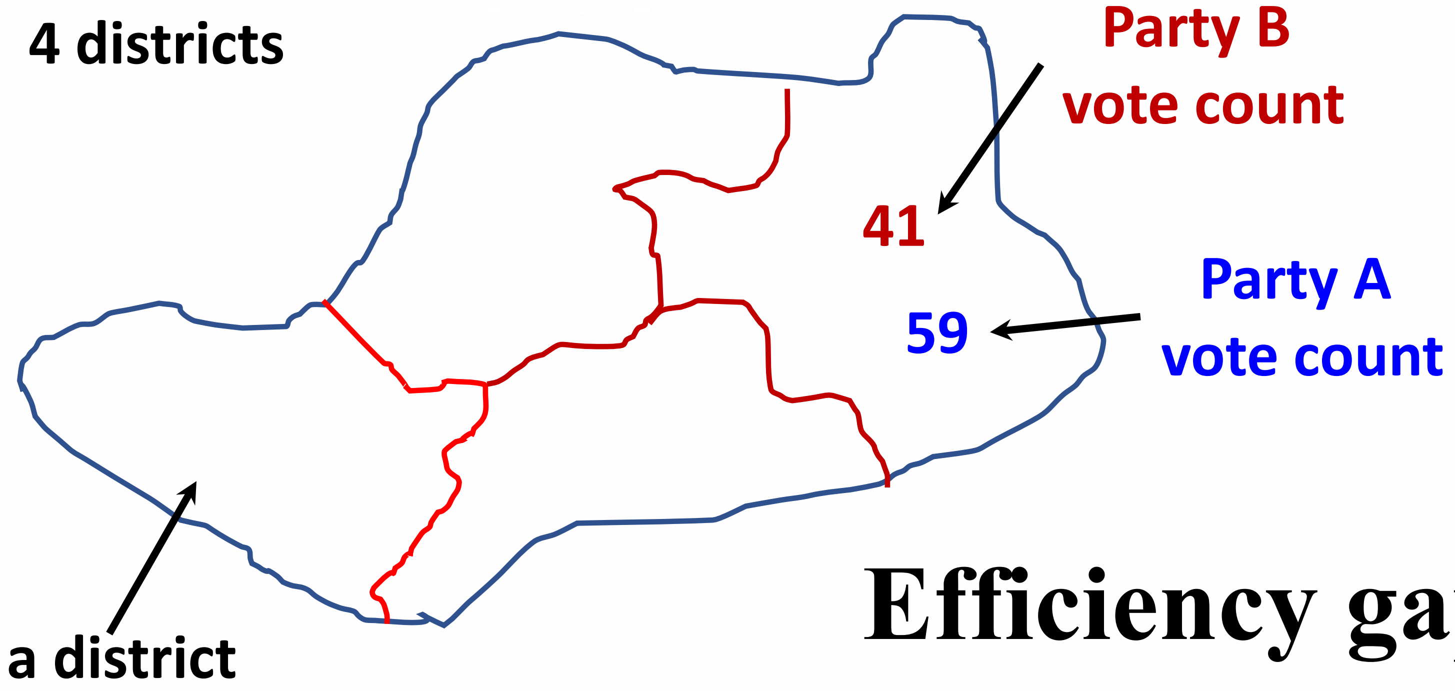
- Introduced by Stephanopoulos and McGhee in 2014 for a 2-party system (such as USA)
- Minimizes absolute difference of total "wasted votes" between the parties
- Very promising in several aspects, e.g.,
  - provides a "mathematically precise" measure of gerrymandering with desirable properties
  - was found legally convincing in a US appeals court case
    - ALAS, Supreme Court overturned the ruling in 2019

“Wasted votes” for a district



- Total votes 100 (need 51 to win)
  - Party A vote 59
  - Party B vote 41
- Wasted votes for Party A 59-51=8
- Wasted votes for Party B 41

“Efficiency gap” measure for the whole map



Efficiency gap

$$= \frac{|\text{sum of Party A wasted votes over all districts} - \text{sum of Party B wasted votes over all districts}|}{\text{Total votes over all districts}}$$

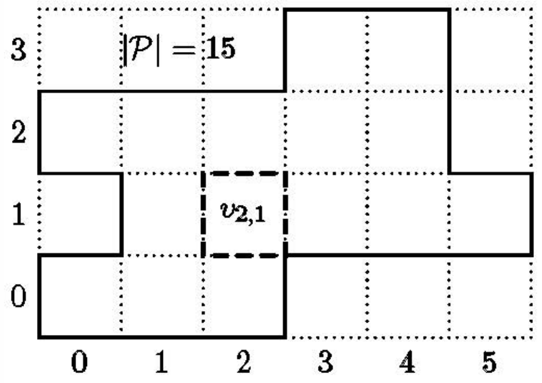
Formalization of the efficiency gap calculation problem

Basic assumption: *only two* parties: Party A and Party B

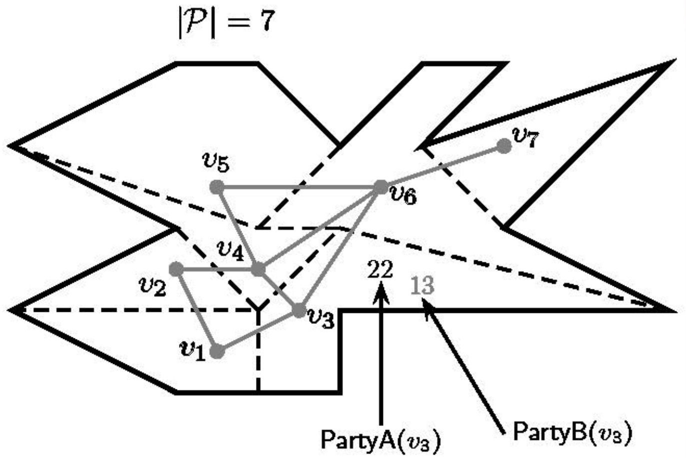
(3^{rd} party votes are negligible, like in USA)

Topological part of an input: a "**map**" $\mathcal{P}$

- ▷ partitioned into *atomic elements* or *cells*
  - *e.g.*, , subdivisions of counties



Rectilinear polygon



Arbitrary polygon

Two possible types of maps:

Rectilinear polygon $\mathcal{P}$ without holes

- ▷ $\mathcal{P}$ placed on a unit grid of size $m \times n$
- ▷ atomic elements (cells) $\Rightarrow$ unit squares of grid inside $\mathcal{P}$
- ▷ $v_{i,j}$: cell on i^{th} row and j^{th} column

Arbitrary polygon $\mathcal{P}$ without holes:

- ▷ atomic elements (cells) $\Rightarrow$ sub-polygons (without holes) inside $\mathcal{P}$
- ▷ *Alternate* way of looking: **planar graph $G(\mathcal{P})$**
  - • nodes are cells
  - • edge connects two cells if they share boundary

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

Parameters of our gerrymandering problem

- **Map $\mathcal{P}$:**
  - *size* $|\mathcal{P}|$: number of cells or nodes in $\mathcal{P}$

- **Cell or node y of $\mathcal{P}$:**
  - $\mathsf{PartyA}(y)$: total number of voters for Party A
  - $\mathsf{PartyB}(y)$: total number of voters for Party B
  - $\mathsf{Pop}(y) = \mathsf{PartyA}(y) + \mathsf{PartyB}(y)$: total number of voters

- **Global:**
  - κ: *required* (legally mandated) number of districts ($1 < \kappa < |\mathcal{P}|$)
    - Hard constraint: solution with different value of κ would be *illegal*
    - precludes designing approximation algorithm in which the value of κ changes even by just ± 1
    - computational hardness for a value of κ may *not* necessarily imply hardness for another value of κ

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

Granularities of numeric parameters

- **Course granularity:**
  - **$\mathsf{Pop}(y)$'s are numbers of arbitrary size**
  - **total number of bits contributes to input size**
  - **data at the "county" level or "census block *group*" level**
- **Fine granularity:**
  - **$\forall$ cell or node y: $\quad 0 < \mathsf{Pop}(y) \leq c$ for some *fixed constant c***
  - **data at the "Voting Tabulation District" (VTD) level or "census block" level**
- **Ultra-fine granularity:**
  - **$\forall$ cell or node y: $\quad \mathsf{Pop}(y) = c$ for some *fixed constant c***
  - **theoretically interesting case, but practically a bit unrealistic**

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

κ	number of districts
$\mathcal{S}$	set of all cells in given polygonal map $\mathcal{P}$ or, set of all nodes in given planar graph $G(\mathcal{P})$
districting scheme	partition of $\mathcal{S}$ into κ subsets $\mathcal{S}_1, \ldots, \mathcal{S}_\kappa$
Notations for each $\mathcal{S}_j$	**Party affiliations in $\mathcal{S}_j$**: $\mathsf{PartyA}(\mathcal{S}_j) = \sum_{y \in \mathcal{S}_j} \mathsf{PartyA}(y)$ $\mathsf{PartyB}(\mathcal{S}_j) = \sum_{y \in \mathcal{S}_j} \mathsf{PartyB}(y)$ **Population of $\mathcal{S}_j$**: $\mathsf{Pop}(\mathcal{S}_j) = \mathsf{PartyA}(\mathcal{S}_j) + \mathsf{PartyB}(\mathcal{S}_j)$

Legal requirements for valid re-districting plans

- ☐ Every $\mathcal{S}_j$ must be a connected polygon
- ☐ Populations of different $\mathcal{S}_j$'s must be as equal as possible

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

Legal requirements for valid re-districting plans

- ☐ Every $\mathcal{S}_j$ must be a connected polygon
- ☐ Populations of different $\mathcal{S}_j$'s must be as equal as possible

  ▷ **Strict partitioning criteria**

  $\{\mathcal{S}_1, \ldots, \mathcal{S}_\kappa\}$ is an *exact* κ*-equipartition* of $\mathcal{S}$, *i.e.*, $\forall\, j : \mathsf{Pop}(\mathcal{S}_j) \in \left\{ \left\lfloor {}^{\mathsf{Pop}(\mathcal{S})}/_{\kappa} \right\rfloor , \left\lceil {}^{\mathsf{Pop}(\mathcal{S})}/_{\kappa} \right\rceil \right\}$

  ▷ **(Multiplicatively) approximate partitioning criteria**

  $\{\mathcal{S}_1, \ldots, \mathcal{S}_\kappa\}$ is a ε*-approximate* κ-equipartition of $\mathcal{S}$, *i.e.*, $\dfrac{\max\left\{\mathsf{Pop}(\mathcal{S}_j)\right\}}{\min\left\{\mathsf{Pop}(\mathcal{S}_j)\right\}} \leq 1 + \varepsilon$

  courts may allow a maximum value of ε in the range of 0.05 to 0.1
  e.g., **, (US Supreme Court ruling in Karcher v. Daggett, 1983)**

  ▷ ***Additively* approximate partitioning criteria**

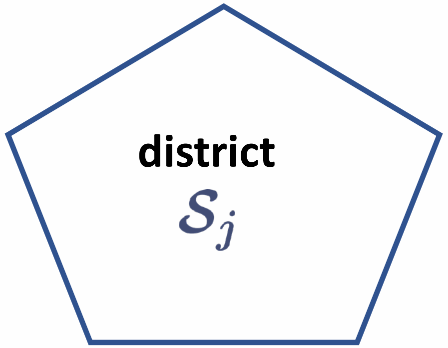
  $\{\mathcal{S}_1, \ldots, \mathcal{S}_\kappa\}$ is an *additive* ε*-approximate* κ-equipartition of $\mathcal{S}$, *i.e.*,

  $\max\left\{\mathsf{Pop}(\mathcal{S}_j)\right\} \leq \min\left\{\mathsf{Pop}(\mathcal{S}_j)\right\} + \varepsilon$

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

"Wasted votes" for a district



- Total votes **100** (Party A needs 50 to win) $\quad \mathsf{Pop}(\mathcal{S}_j)$
  - Party A vote **59** $\quad \mathsf{PartyA}(\mathcal{S}_j)$
  - Party B vote **41** $\quad \mathsf{PartyB}(\mathcal{S}_j)$
- Wasted votes for Party A $\quad$ 59 – 50 = **9** $\quad \mathsf{PartyA}(\mathcal{S}_j) - \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j)$
- Wasted votes for Party B $\quad$ **41** $\quad \mathsf{PartyB}(\mathcal{S}_j)$
- Efficiency gap for $\mathcal{S}_j$ $\quad$ 9 – 41 = - **32**

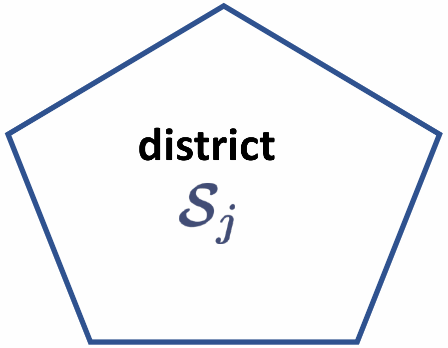
$$\mathsf{Effgap}(\mathcal{S}_j) = \begin{cases} \left(\mathsf{PartyA}(\mathcal{S}_j) - \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j)\right) - \mathsf{PartyB}(\mathcal{S}_j) & \text{if } \mathsf{PartyA}(\mathcal{S}_j) \geq \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j) \\ = 2\mathsf{PartyA}(\mathcal{S}_j) - \frac{3}{2}\mathsf{Pop}(\mathcal{S}_j) & \end{cases}$$

from the point of view of Party A (the victim party of gerrymandering)

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

“Wasted votes” for a district



- **Total votes 100 (Party A needs 50 to win)** $\quad \mathsf{Pop}(\mathcal{S}_j)$
  - **Party A vote 41** $\quad \mathsf{PartyA}(\mathcal{S}_j)$
  - **Party B vote 59** $\quad \mathsf{PartyB}(\mathcal{S}_j)$
- **Wasted votes for Party A 41** $\quad \mathsf{PartyA}(\mathcal{S}_j)$
- **Wasted votes for Party B 59 - 50 = 9** $\quad \mathsf{PartyB}(\mathcal{S}_j) - \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j)$
- **Efficiency gap for** $\mathcal{S}_j$ **41 - 9 = - 32**

$$\mathsf{Effgap}(\mathcal{S}_j) = \begin{cases} \mathsf{PartyA}(\mathcal{S}_j) - \left(\mathsf{PartyB}(\mathcal{S}_j) - \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j)\right) \\ = 2\mathsf{PartyA}(\mathcal{S}_j) - \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j) \end{cases} \quad \text{if } \mathsf{PartyA}(\mathcal{S}_j) < \frac{1}{2}\mathsf{Pop}(\mathcal{S}_j)$$

from the point of view of Party A (the victim party of gerrymandering)

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

κ	number of districts
$\mathcal{S}$	set of all cells in given polygonal map $\mathcal{P}$ or, set of all nodes in given planar graph $G(\mathcal{P})$
districting scheme	partition of $\mathcal{S}$ into κ subsets $\mathcal{S}_1, \ldots, \mathcal{S}_\kappa$

$$\textsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa) = \left| \sum_{j=1}^{\kappa} \textsf{Effgap}(\mathcal{S}_j) \right|$$

(to be minimized)

from the point of view of Party A (the victim party of gerrymandering)

Formalization of the efficiency gap calculation problem

only two parties: Party A and Party B

κ-district Minimum Wasted Vote Problem (MIN-WVP$_\kappa$)

Input

- ▷ map $\mathcal{P}$ with $\mathsf{Pop}(y)$, $\mathsf{PartyA}(y)$, $\mathsf{PartyB}(y)$ for every cell $y \in \mathcal{P}$
- ▷ integer $1 < \kappa \leq |\mathcal{P}|$

Assumption $\mathcal{P}$ has at least one κ-equipartition why this assumption ?

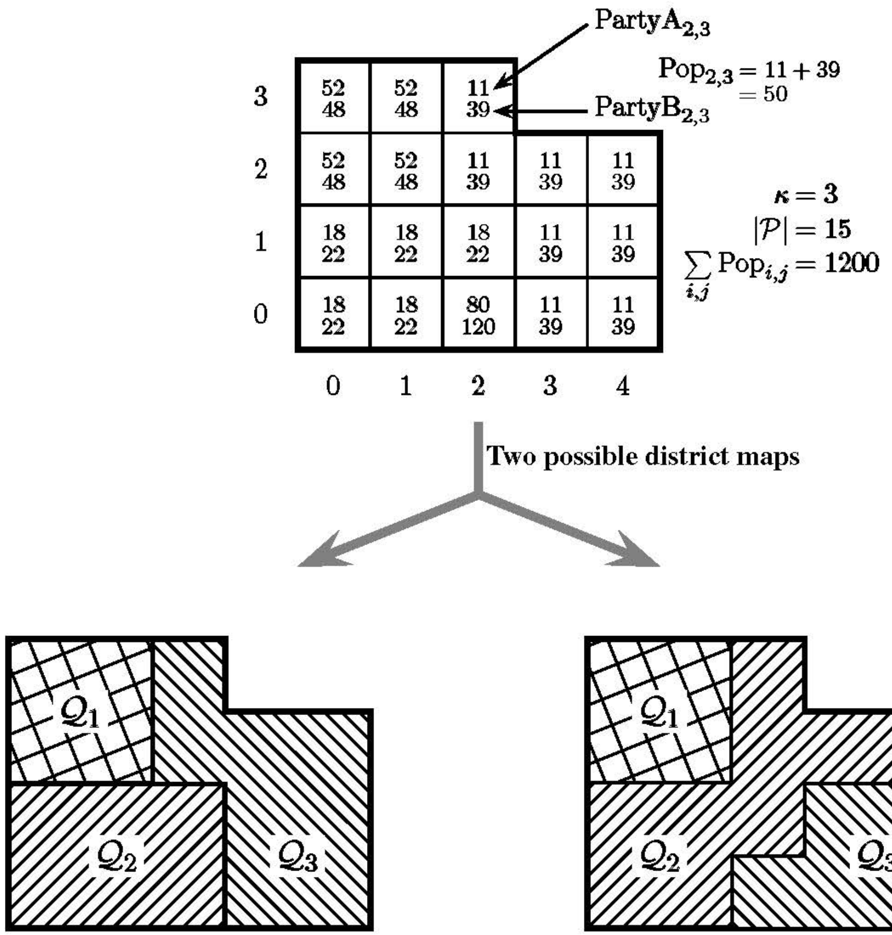
Valid solution Any κ-equipartition $\mathcal{S}_1, \ldots, \mathcal{S}_\kappa$ of $\mathcal{P}$ ‡

Objective *minimize* $\mathsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa) = \left| \sum_{j=1}^{\kappa} \mathsf{Effgap}(\mathcal{S}_j) \right|$

Notation $\mathsf{OPT}_\kappa(\mathcal{P}) \stackrel{\text{def}}{=} \min \left\{ \mathsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa) \mid \mathcal{S}_1, \ldots, \mathcal{S}_\kappa \text{ is a } \kappa\text{-equipartition of } \mathcal{P} \right\}$

‡ in exact or approximate sense

A numerical example to illustrate efficiency gap calculation problem



	PartyA($\mathcal{Q}_{...}$)	PartyB($\mathcal{Q}_{...}$)	Effgap($\mathcal{Q}_{...}$)
$\mathcal{Q}_1$	208	192	−184
$\mathcal{Q}_2$	170	230	140
$\mathcal{Q}_3$	88	312	−24

$\text{Effgap}(\mathcal{P}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3) = |-184 + 140 - 24| = 68$

	PartyA($\mathcal{Q}_{...}$)	PartyB($\mathcal{Q}_{...}$)	Effgap($\mathcal{Q}_{...}$)
$\mathcal{Q}_1$	208	192	−184
$\mathcal{Q}_2$	134	266	58
$\mathcal{Q}_3$	124	276	48

$\text{Effgap}(\mathcal{P}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3) = |-184 + 58 + 48| = 78$

Mathematical properties of $\mathsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa)$: set of attainable values

assume *strict* κ-equipartition, *i.e.*, $\mathsf{Pop}(\mathcal{S}_1) = \cdots = \mathsf{Pop}(\mathcal{S}_\kappa)$

Lemma 1

- $\mathsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa)$ assumes one of the $\kappa + 1$ values:

$$\left| 2 \times \mathsf{PartyA}(\mathcal{P}) - \left(z + \frac{\kappa}{2}\right) \frac{\mathsf{Pop}(\mathcal{P})}{\kappa} \right| \quad \text{for } z = 0, 1, \ldots, \kappa$$

- If $\mathsf{Effgap}_\kappa(\mathcal{P}, \mathcal{S}_1, \ldots, \mathcal{S}_\kappa) = \left| 2\,\mathsf{PartyA}(\mathcal{P}) - \left(z + \frac{\kappa}{2}\right) \frac{\mathsf{Pop}(\mathcal{P})}{\kappa} \right|$ then

$$\frac{\mathsf{Pop}(\mathcal{P})}{2\,\kappa} z \leq \mathsf{PartyA}(\mathcal{P}) \leq \frac{\mathsf{Pop}(\mathcal{P})}{2\,\kappa} z + \frac{1}{2} \mathsf{Pop}(\mathcal{P})$$

Illustrative example: $\kappa = 2$

only 3 possible values of $\mathsf{Effgap}_2(\mathcal{P}, \mathcal{S}_1, \mathcal{S}_2)$

$$\left| 2 \times \mathsf{PartyA}(\mathcal{P}) - \tfrac{1}{2}\,\mathsf{Pop}(\mathcal{P}) \right| \quad \text{or} \quad \left| 2 \times \mathsf{PartyA}(\mathcal{P}) - \mathsf{Pop}(\mathcal{P}) \right| \quad \text{or} \quad \left| 2 \times \mathsf{PartyA}(\mathcal{P}) - \tfrac{3}{2}\,\mathsf{Pop}(\mathcal{P}) \right|$$

First a “somewhat” bad news
(worst-case computational complexity meets gerrymandering)

Theorem (informal description)

Not only calculation of efficiency gap is NP-complete, but

assuming P $\neq$ NP, no non-trivial approximation is possible in polynomial time

But, have no fear !
We have only shown hardness
in theoretical *worst-case*

Worst-case computational complexity meets gerrymandering

Assumptions

- ☐ Map $\mathcal{P}$: rectilinear polygon without holes
- ☐ Strict partitioning criteria: $\{\mathcal{S}_1, \ldots, \mathcal{S}_\kappa\}$ is exact κ-*equipartition* of $\mathcal{S}$
- ☐ Course granularity: $\mathsf{Pop}(y)$'s are numbers of arbitrary size
- ☐ $\mathrm{P} \neq \mathsf{NP}$

Theorem 1

For any rational constant $\varepsilon \in (0,1)$, for any ρ and all $2 \leq \kappa \leq \varepsilon|\mathcal{P}|$,
$\textsc{Min-wvp}_\kappa$ problem for rectilinear polygon $\mathcal{P}$
does not admit a ρ-approximation algorithm

Reduction: from PARTITION problem

Worst-case computational complexity meets gerrymandering

Assumptions

- Map $\mathcal{P}$: planar graph $G = (V, E)$
- (Multiplicatively) approximate partitioning criteria:

  $\{\mathcal{S}_1, \dots, \mathcal{S}_\kappa\}$ is a ε-*approximate* κ-equipartition of $\mathcal{S}$, *i.e.*, $\dfrac{\max\{\mathsf{Pop}(\mathcal{S}_j)\}}{\min\{\mathsf{Pop}(\mathcal{S}_j)\}} \leq 1 + \varepsilon$
- Fine granularity: $\forall$ node y: $0 < \mathsf{Pop}(y) \leq c$ for some *fixed constant* c

Theorem 2

For any constant $0 < \varepsilon < 1/2$,
computing an exact solution of the $\textsc{Min-wvp}_\kappa$ problem
is NP-complete

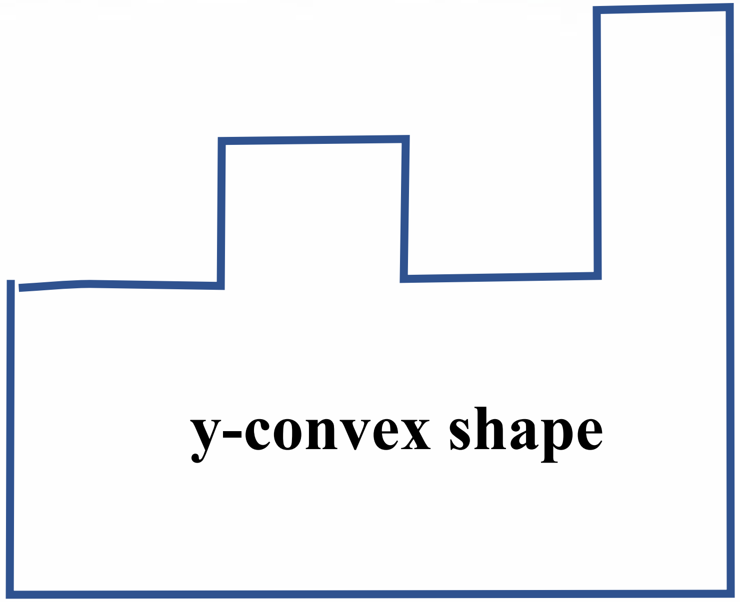
Proof does *not* provide any non-trivial inapproximability ratio

Reduction: from maximum independent set for planar cubic graphs

However, even in theory, we can efficiently compute efficiency gap under "*reasonable*" assumptions

e.g., with these assumptions:

- Input map: a rectilinear polygon $\mathcal{P}$ (without holes)
- Every district must have a "nice" shape (*y-convex* shape)
- κ (number of districts) is *constant*
- Total population Pop($\mathcal{P}$) is *polynomial* in number of cells $|\mathcal{P}|$



We developed and implemented a simple heuristic algorithm based on "local search" method

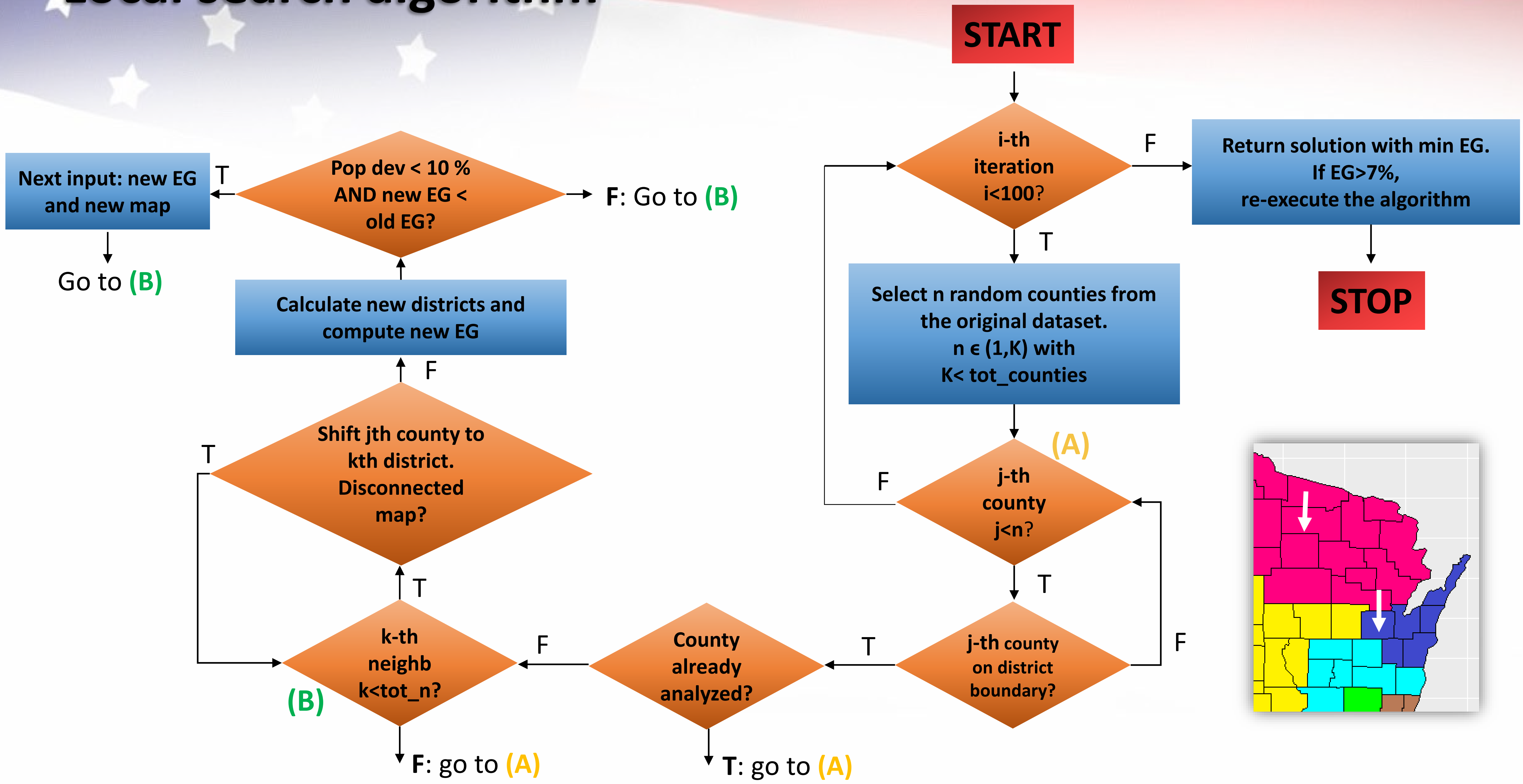
- Start with some existing or random valid solution

- Search for nearby valid solutions by randomly "swapping" local regions among various districts

    - Pitfall: can get stuck with far-away local optima
      but, does not seem to often occur for real maps

➢ **Next few slides: results for real maps**

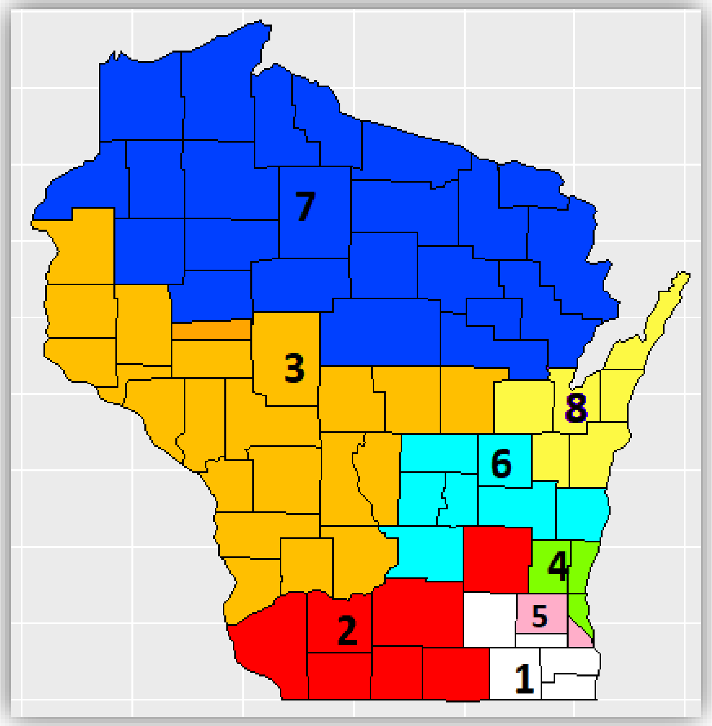
Local search algorithm



Wisconsin

Total votes: 2,841,407
Dem votes: 1,441,804 ~ *51%*
Rep votes: 1,399,603 ~ *49%*

Current EG: 14.8%
Dem #seats: 3
Rep #seats: 5

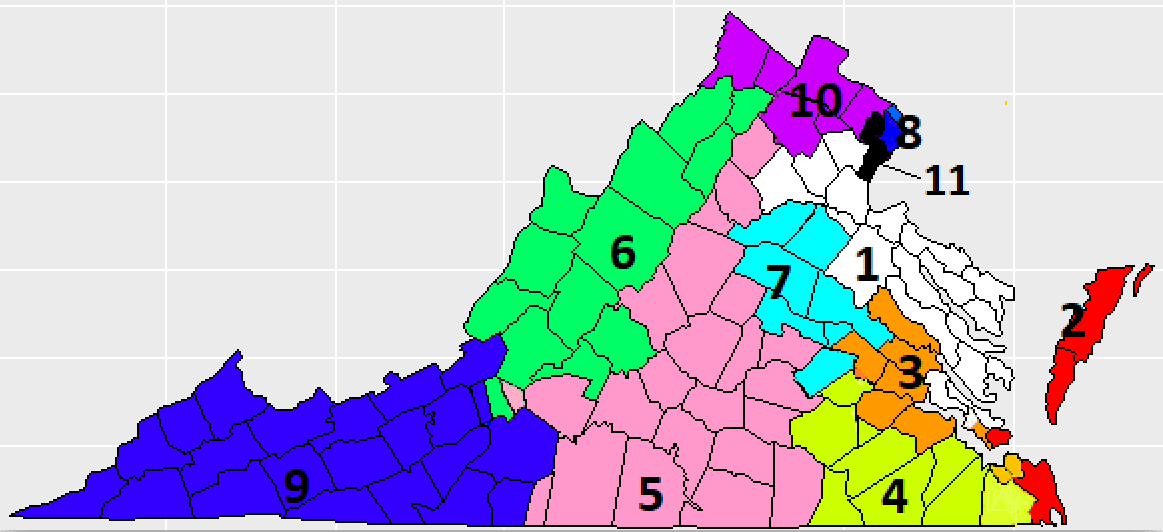


New EG: 3.8%
Dem #seats: 3
Rep #seats: 5

Virginia

Total votes: 3,569,498
Dem votes: 1,736,164 ~ *49%*
Rep votes: 1,833,334 ~ *51%*

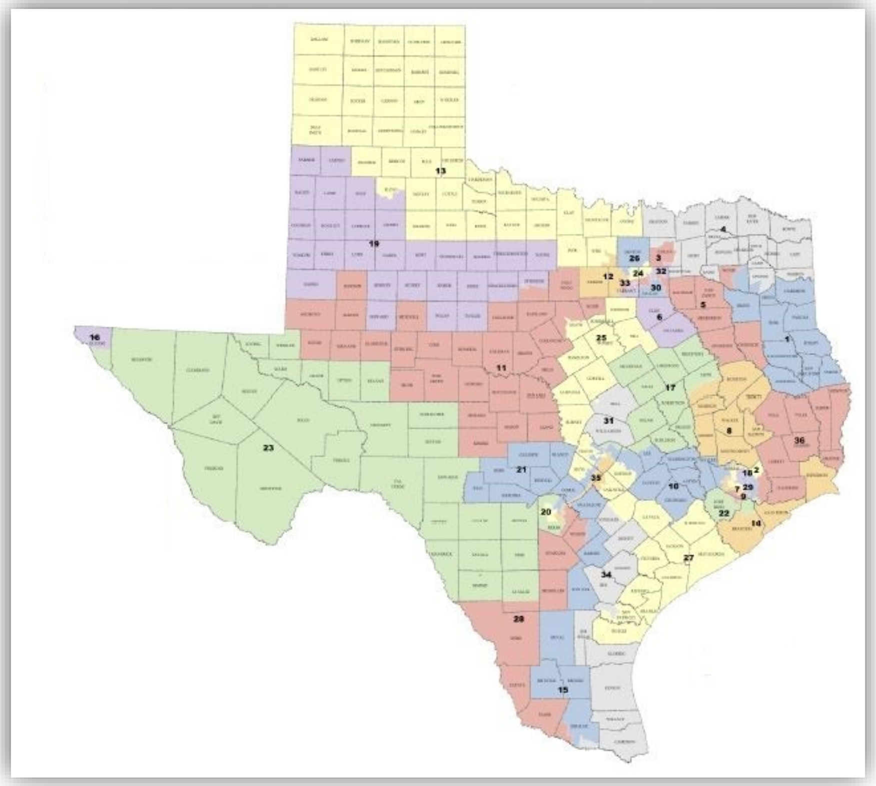
Current EG: 22%
Dem #seats: 3
Rep #seats: 8



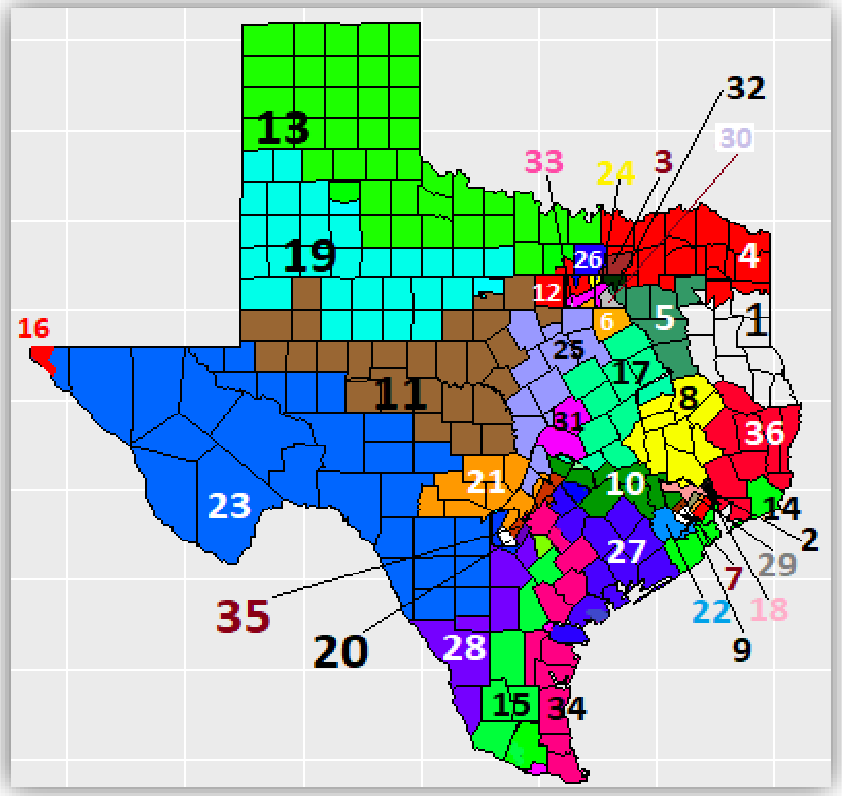
New EG: 3.6%
Dem #seats: 5
Rep #seats: 6

Texas

Total votes: 7,379,170
Dem votes: 2,949,900 ~ *40%*
Rep votes: 4,429,270 ~ *60%*



Current EG: 4.01%
Dem #seats: 12
Rep #seats: 24

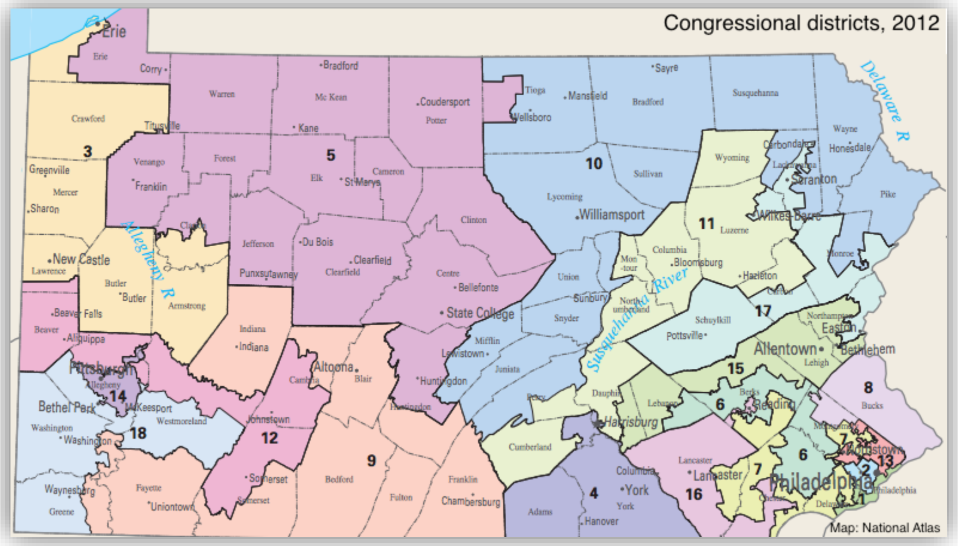


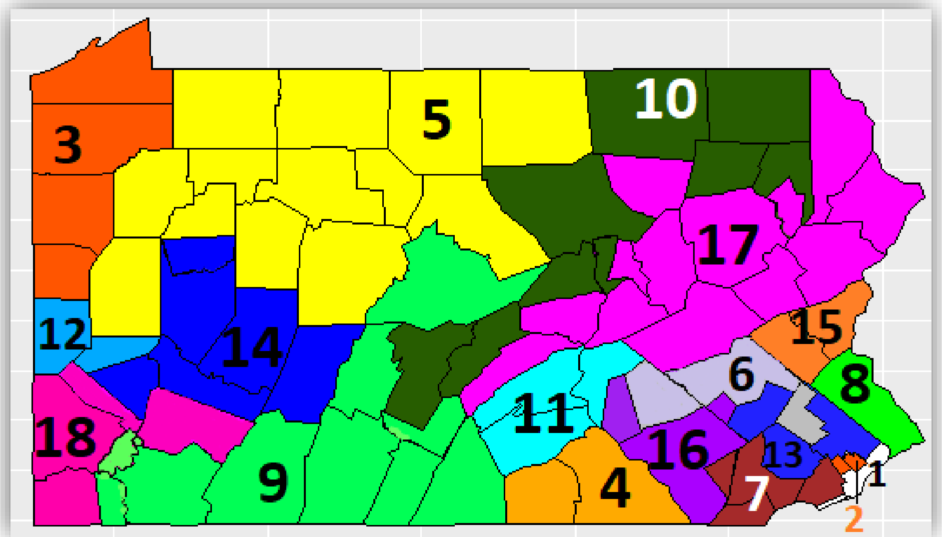
New EG: 3.3%
Dem #seats: 12
Rep #seats: 24

Pennsylvania

2012 House Elections

Total votes: 5,374,461
Dem votes: 2,722,560 ~ *51%*
Rep votes: 2,651,901 ~ *49%*





Current EG: 23.8%
Dem #seats: 5
Rep #seats: 13

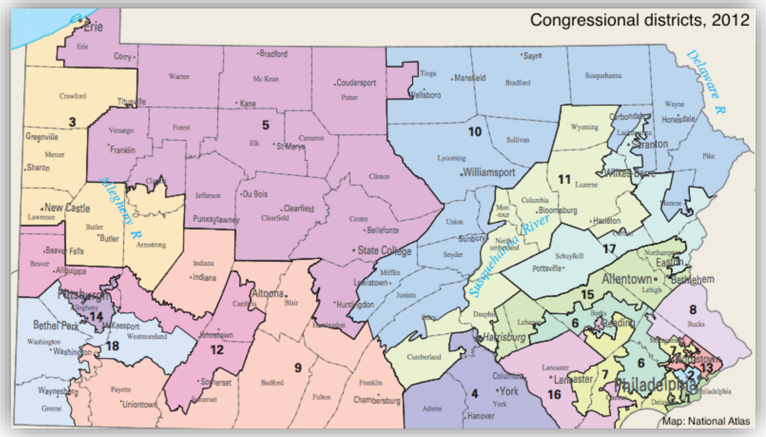
New EG: 8.64%
Dem #seats: 6
Rep #seats: 12

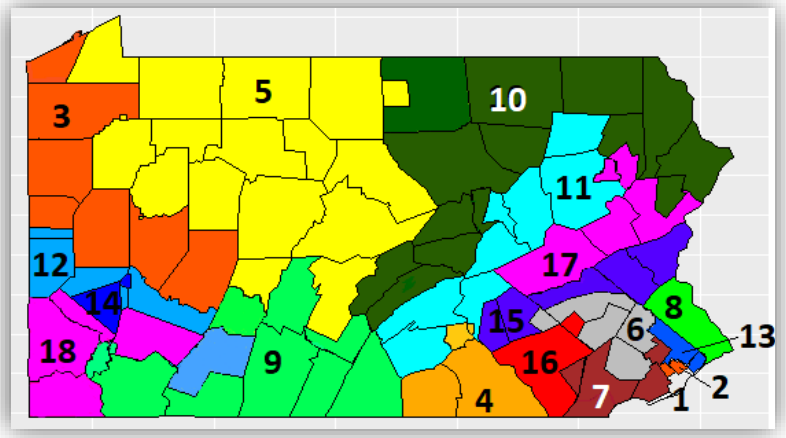
Pennsylvania

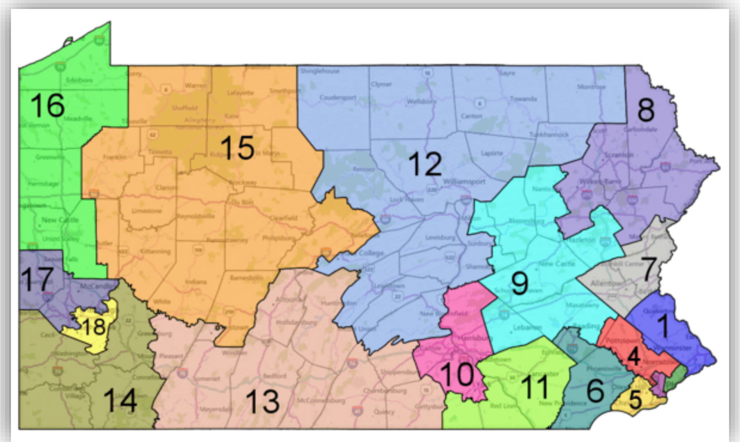
2016 Presidential Elections

Total votes: 5,896,628
Dem votes: 2,925,776 ~ *50%*
Rep votes: 2,970,852 ~ *50%*

Created on Feb. 2018 (by a local court in PA) and based on symmetry between seat share and vote share.







Current EG: 14.34%
Dem #seats: 6
Rep #seats: 12

New EG: 8.05%
Dem #seats: 7
Rep #seats: 11

New EG: 3%
Dem #seats: 8
Rep #seats: 10

Some Interesting Insights based on simulation results

- **Seat gain vs. efficiency gap**
  - lower efficiency gap does not necessarily lead to seat gains for the loosing party
- **Compactness vs. efficiency gap**
  - Our new district map have fewer districts that are oddly shaped compared to the current gerrymandered maps
- **How natural are current gerrymandered districts?**
  - It seems that original gerrymandered districts are far from being a product of arbitrarily random decisions

Future research

Science of gerrymandering is a huge garden with so many unknown fruits for hungry theoretical computer scientists !

So many questions, so few answers

- **Define and analyze new quantitative measures of gerrymandering**
  - **What about 3 or more party systems ?**

- **Analyze computational complexities of existing measures of gerrymandering**

- **Join court cases as an expert witness and convince judges that computational complexity matters**

Journal paper: https://link.springer.com/article/10.1007/s10878-020-00589-x

Data files: https://www.cs.uic.edu/~dasgupta/gerrymander/

Thank you for your attention!

Questions?