Hard and Easy Instances of L-Tromino Tilings [1]

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October 13, 2020

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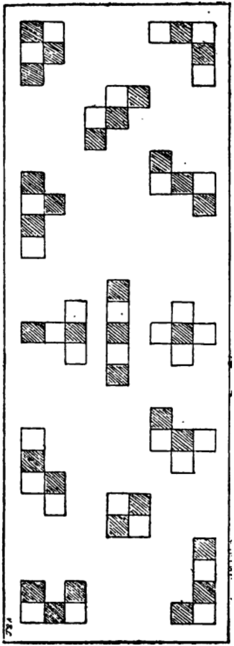
In proceedings of 13th WALCOM, February 27 - March 02, 2019.
arXiv:1710.04640.

Outline

1 Introduction



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Polyominoes

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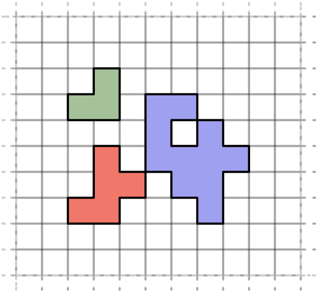

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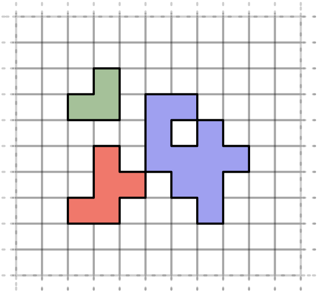

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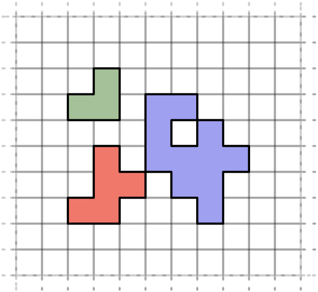

- Every cell (square) is fixed in a square lattice.
- Two cell are adjacent if the Manhattan distance is 1.

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L-Tromino Tiling Problem

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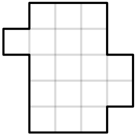
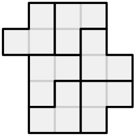


(a) A region R



(b) A tiling of region R

L-Tromino Tiling Problem (cont'd)

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T. Horiyama, T. Ito, K. Nakatsuka, A. Suzuki and R. Uehara (2012) constructed a one-one reduction from 1-in-3 SAT.

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Aztec Rectangle

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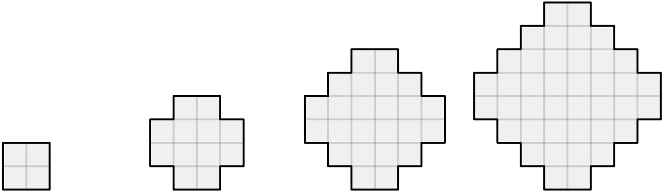

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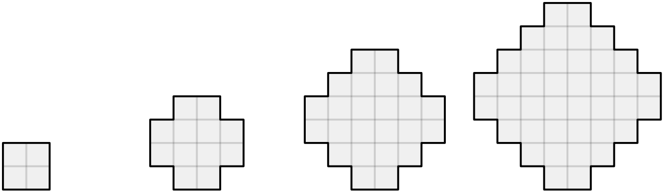

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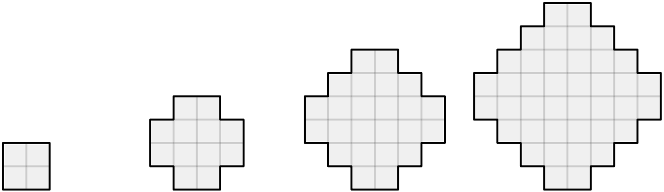

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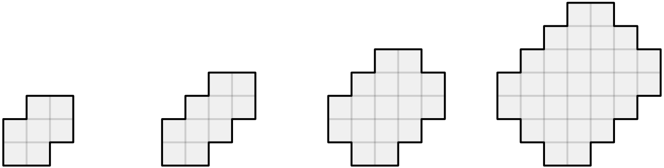

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Tiling Aztec Rectangle (cont'd)

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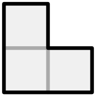
Each piece of L-tromino **covers 3 cells**.



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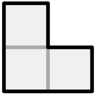

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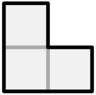

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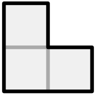

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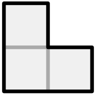

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Theorem

An Aztec rectangle $\mathcal{AR}_{a,b}$ has a tiling with L-trominoes
$\iff |\mathcal{AR}_{a,b}| \equiv 0 \pmod 3$
$\iff (a,b)$ *is equal to* $(3k, 3k')$ *or* $(3k-1, 3k'-1)$ *for some* $k, k' \in \mathbb{N}$.

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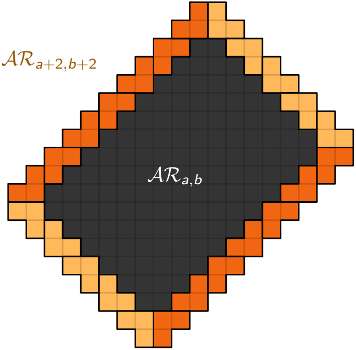

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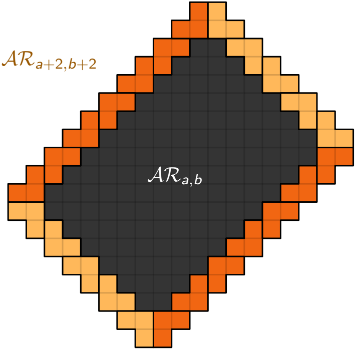

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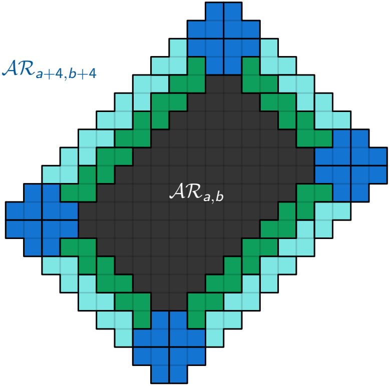

(b) Pattern 2

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- If (a, b) equals $(3, 3k')$, use pattern 3.
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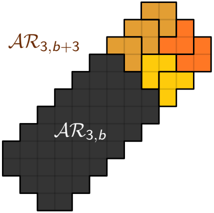

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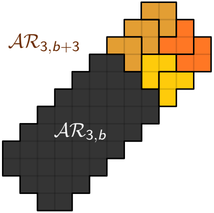

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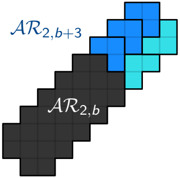

(b) Pattern 4

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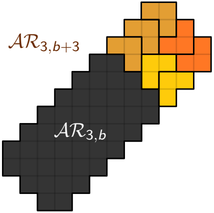

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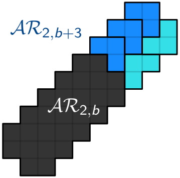

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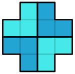
Base case: $\mathcal{AR}_{2,2}$ and $\mathcal{AR}_{3,3}$.



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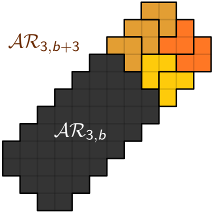

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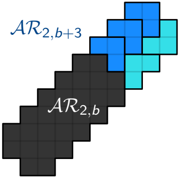

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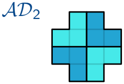

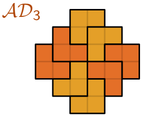

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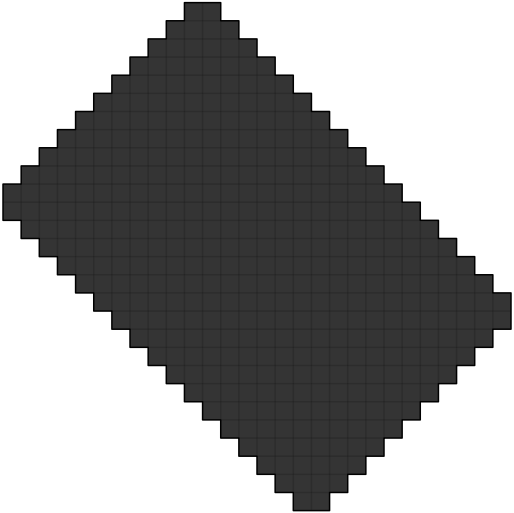

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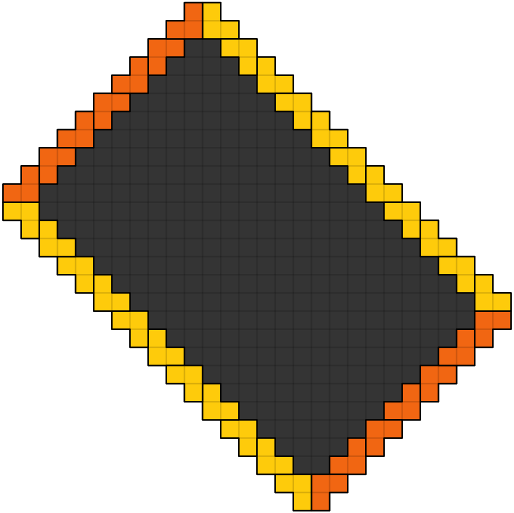

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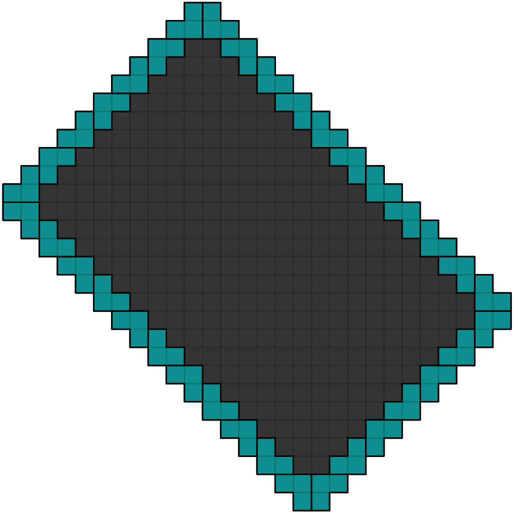

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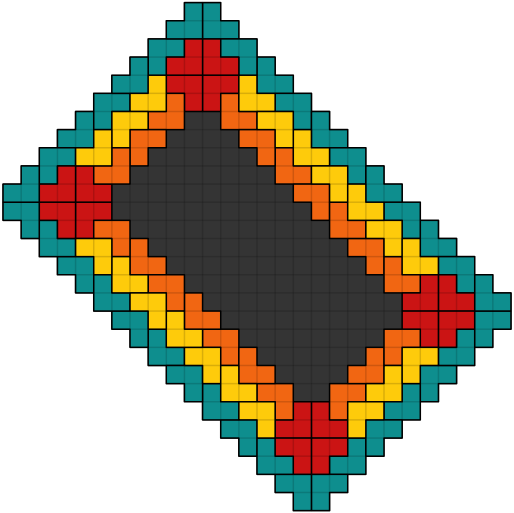

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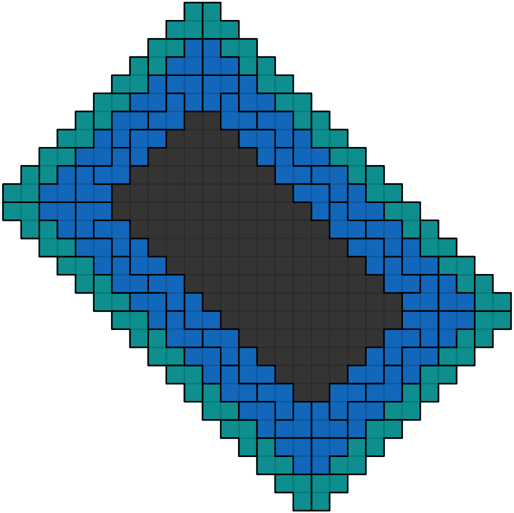

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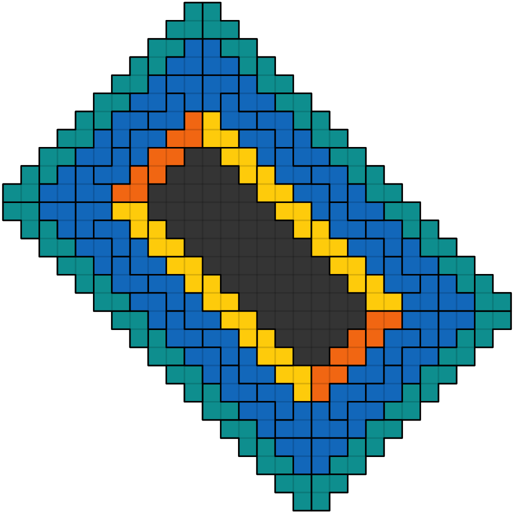

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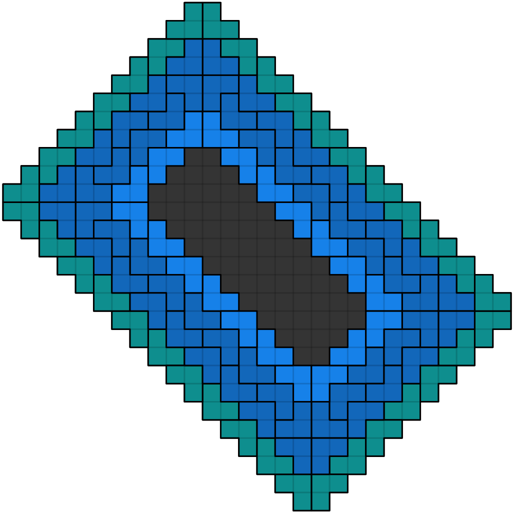

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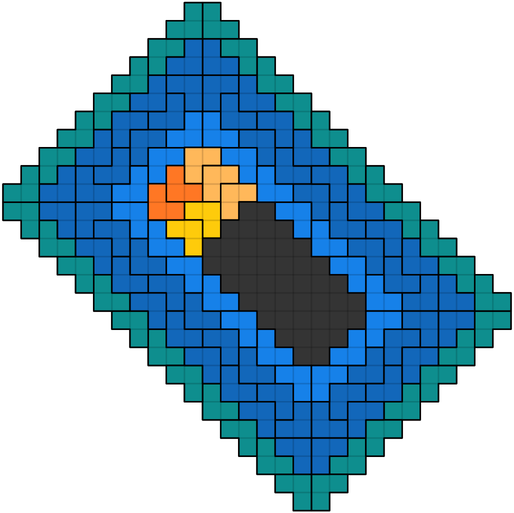

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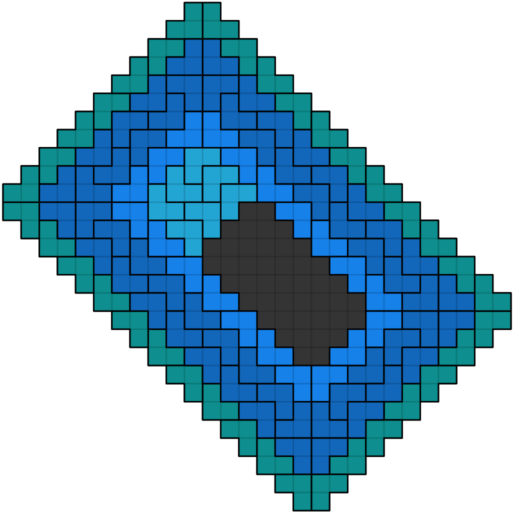

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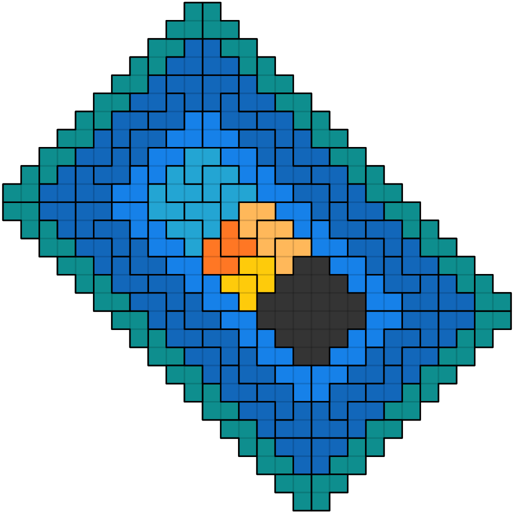

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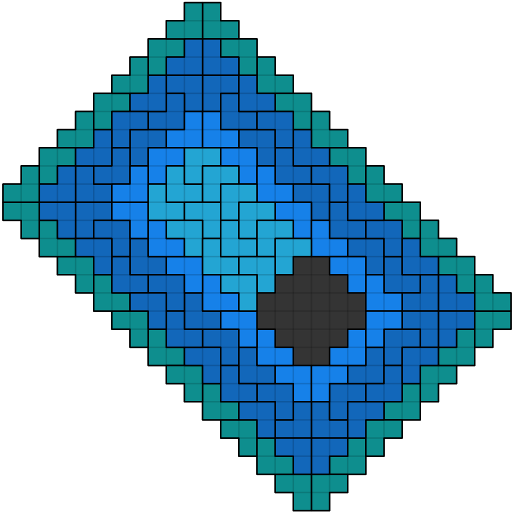

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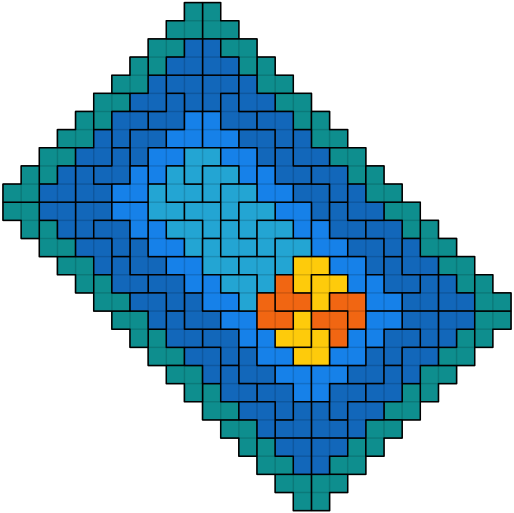

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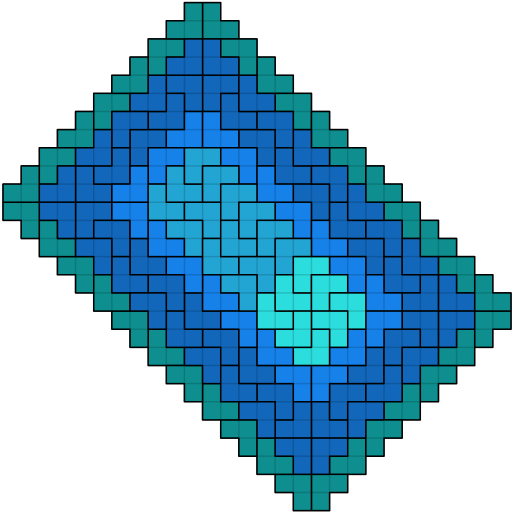

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Tiling Aztec Rectangle with a single defect

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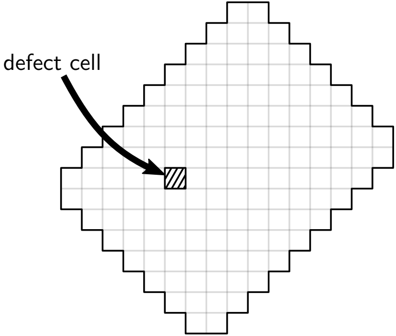

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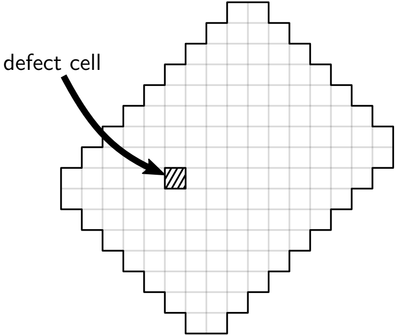
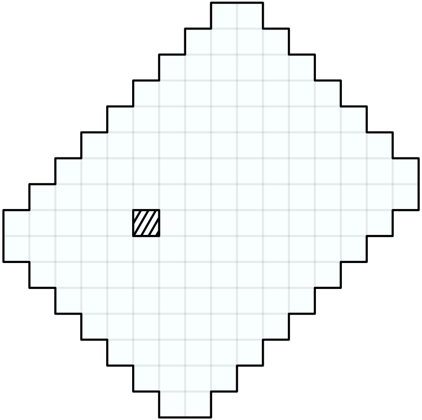


Theorem

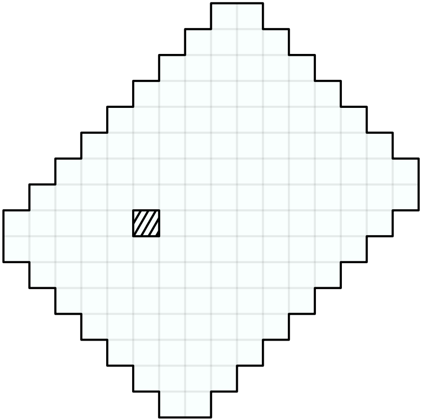
An Aztec rectangle $\mathcal{AR}_{a,b}$ *with one defect has a tiling with L-trominoes*
$\iff$ $|\mathcal{AR}_{a,b}| \equiv 1 \pmod 3$
$\iff$ *a or b is equal to* $3k - 2$ *for some* $k \in \mathbb{N}$.

Tiling Aztec Rectangle with a single defect (cont'd)



Tiling Aztec Rectangle with a single defect (cont'd)

- Place a *fringe* where it covers the defect.



Tiling Aztec Rectangle with a single defect (cont'd)

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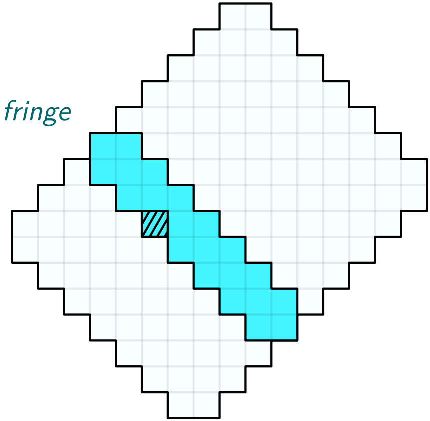

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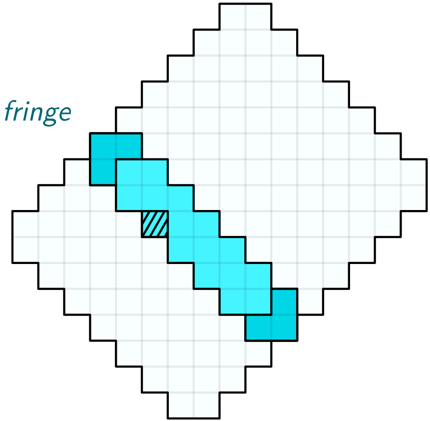

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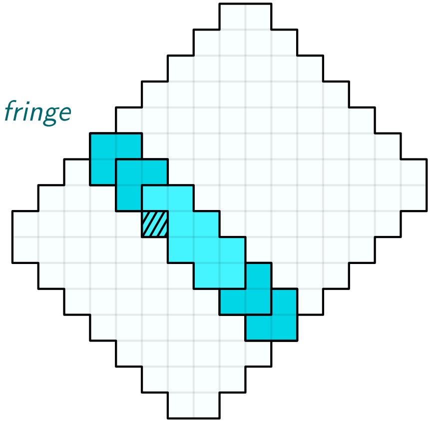

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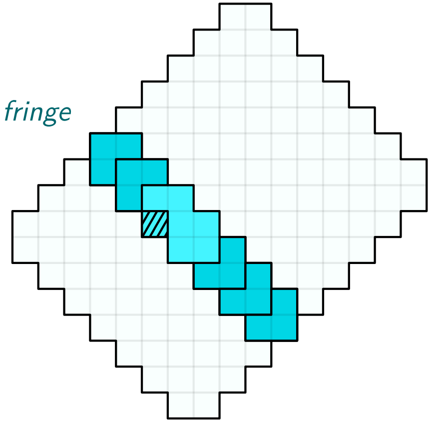

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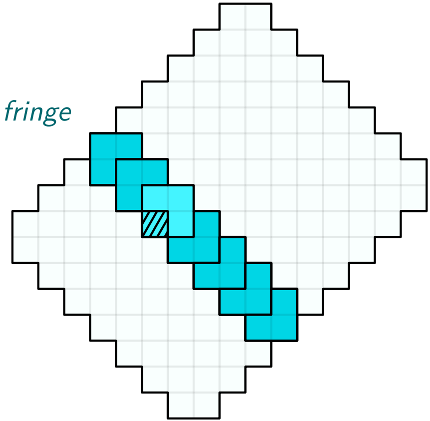

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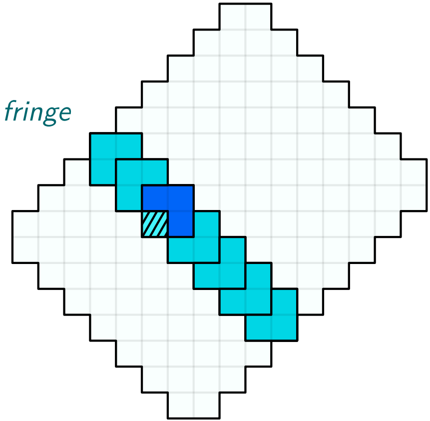

Tiling Aztec Rectangle with a single defect (cont'd)

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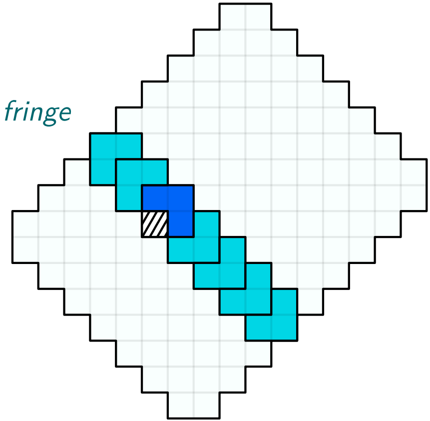

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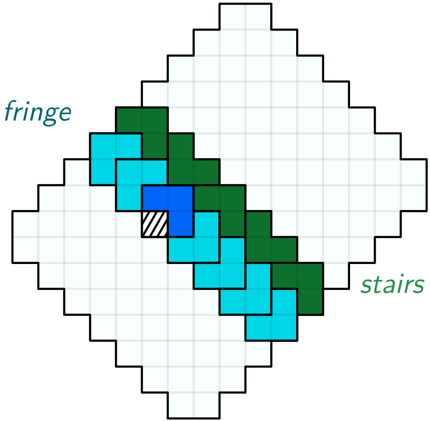

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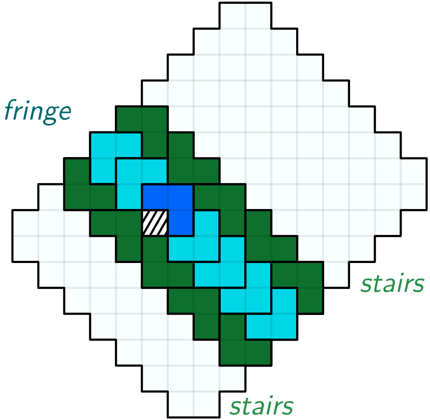

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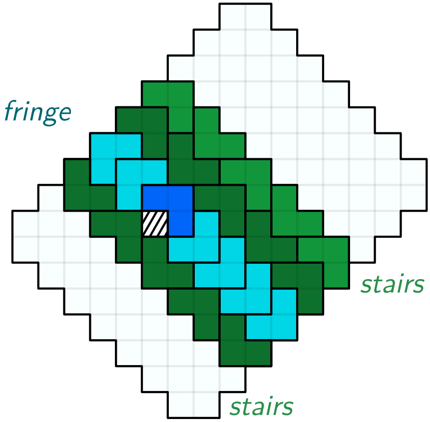

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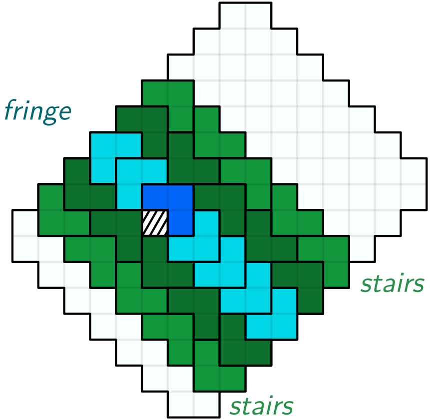

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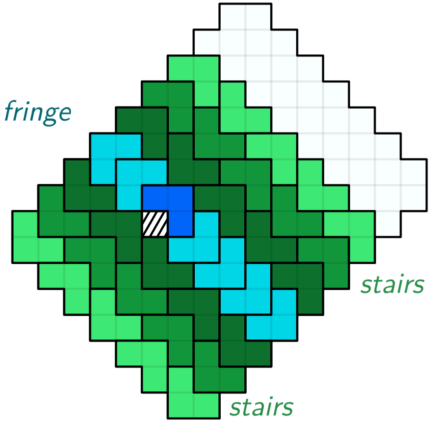

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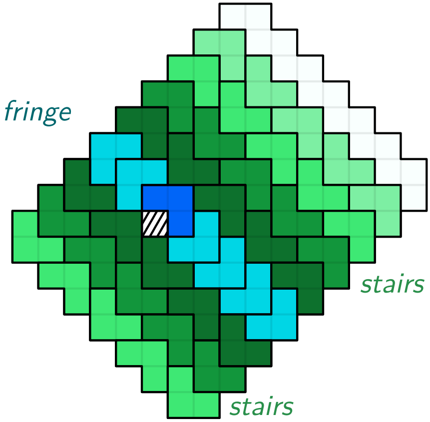

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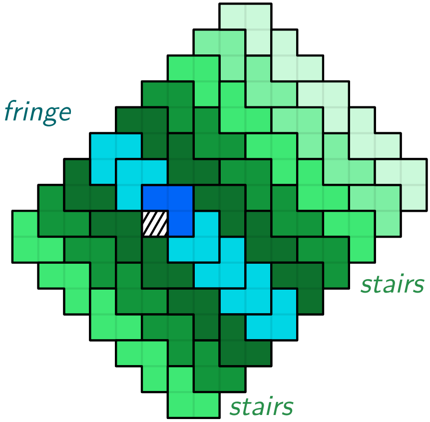

Outline

Tiling Aztec Rectangle with an unbounded number of defects

Tiling Aztec Rectangle with an unbounded number of defects

Theorem

The problem of tiling Aztec Rectangle $\mathcal{AR}_{a,b}$ with an unbounded number of defects is ***NP-complete****.*

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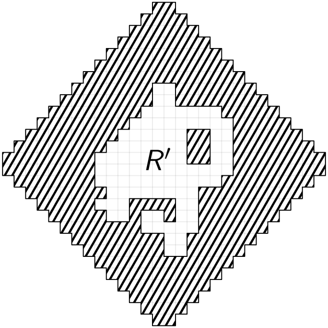

3 180-Tromino Tiling

Outline

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The 180-tromino tiling problem only allows 180° rotations of L-trominoes, i.e., the tile set can be

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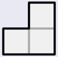
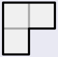
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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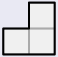
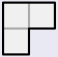
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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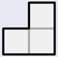
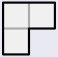
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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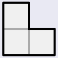
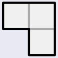
$\Sigma = \{$ **left-oriented** 180-trominoes $\} = \{$ ,  $\}$.

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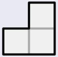
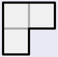
$\Sigma = \{$ **right-oriented** 180-trominoes $\} = \{$ ,  $\}$

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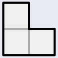
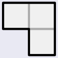
$\Sigma = \{$ **left-oriented** 180-trominoes $\} = \{$ ,  $\}$.

With no loss of generality, we will only consider **right-oriented 180-trominoes**.

180°L-Tromino Tiling (cont'd)

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Theorem (Conway y Lagarias, 1990)

There is a one-one correspondence between 180-tromino tiling and the triangular trihex tiling.

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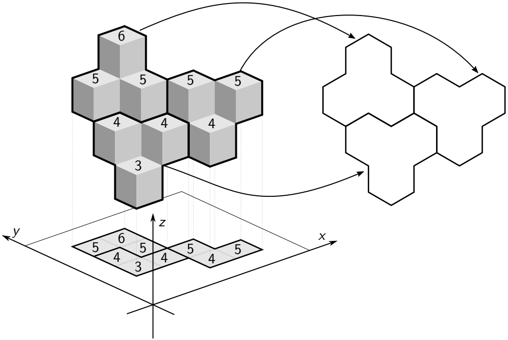

Transformation from **triangular trihex** to **180-tromino**

Definition

A **cell tetrasection** is a division of a cell into 4 equal size cells.

180°L-Tromino Tiling (cont'd)

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A **cell tetrasection** is a division of a cell into 4 equal size cells.

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A **tetrasected polyomino** $P^{\boxplus}$ is obtained by tetrasecting each cell of a poylomino P.

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If there is a I-tromino tiling for some R, then there is also a 180-tromino tiling for $R^{\boxplus}$.

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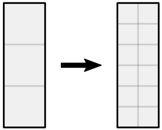

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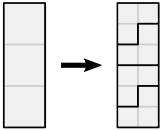

180°L-Tromino Tiling (cont'd)

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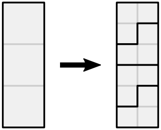

However, it is not known if the converse statement is true or false.

180°L-Tromino Tiling (cont'd)

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Horiyama et al. proved that the l-tromino tiling problem is **NP-Complete**.

180°L-Tromino Tiling (cont'd)

Horiyama et al. proved that the I-tromino tiling problem is **NP-Complete**.

Theorem (Horiyama *et al.*, 2012)

1-IN-3 SAT $\leq_p$ I-TROMINO

180°L-Tromino Tiling (cont'd)

Horiyama et al. proved that the I-tromino tiling problem is **NP-Complete**.

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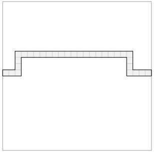
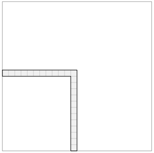
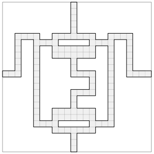


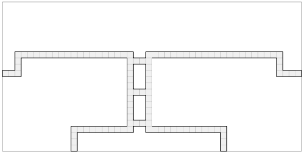
(a) Line gadget.



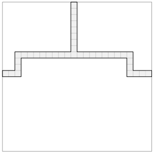
(b) Corner gadget.



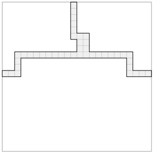
(c) Cross gadget.



(d) Duplicator gadget.



(e) Clause gadget.



(f) Negated clause gadget.

180°L-Tromino Tiling (cont'd)

In each gadget G, I-tromino tiling for G can be simulated with 180-tromino tiling for $G^{\boxplus}$.

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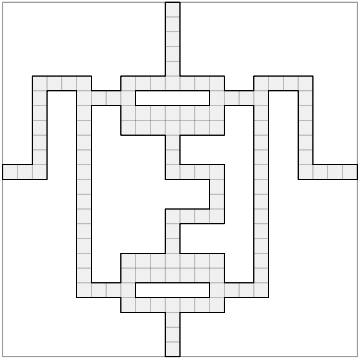

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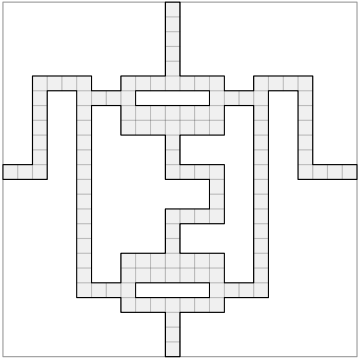

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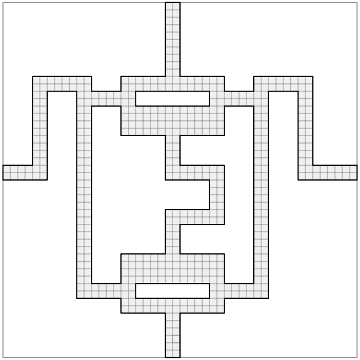

(b) Tetrasected gadget $G^{\boxplus}$.

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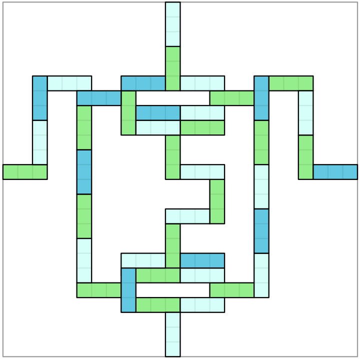

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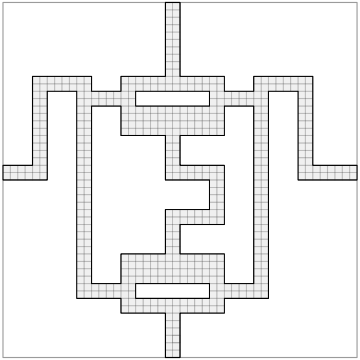

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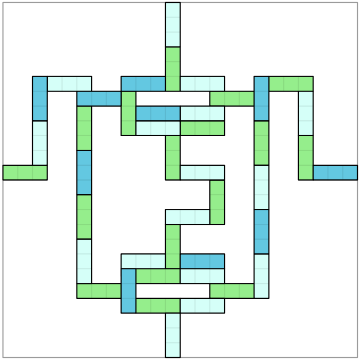

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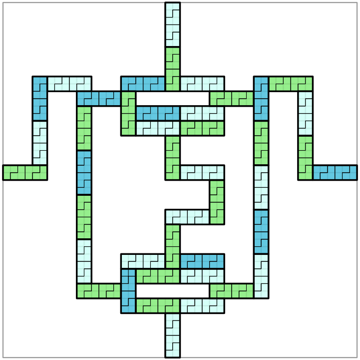

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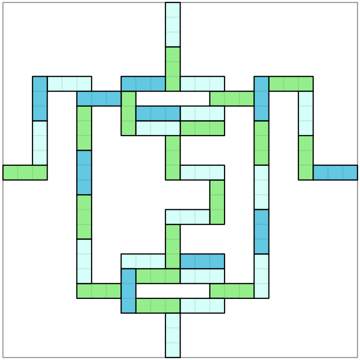

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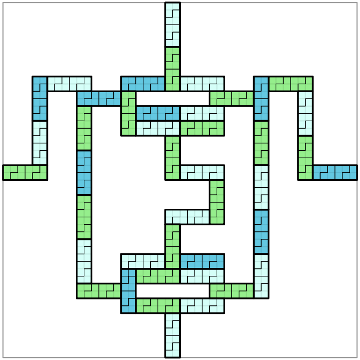

(b) Tetrasected gadget $G^{\boxplus}$.

Theorem

180-tromino tiling is ***NP-complete****.*

Outline

1. Introduction
   - Polyominoes
   - L-Tromino Tiling Problem
2. Tiling Aztec Rectangles
   - Aztec Rectangle
   - Aztec Rectangle with a single defect
   - Tiling Aztec Rectangle with unbounded number of defects
3. 180-Tromino Tiling
   - A rotation constraint
   - Forbidden Polyominoes
4. Tiling Tetrasected Region
   - Special case: the Dual Graph is a Tree
   - General case

The 180-tromino tiling can also be reduced to the **Maximum Independent Set** problem.

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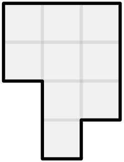
R



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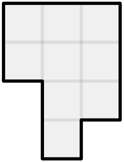

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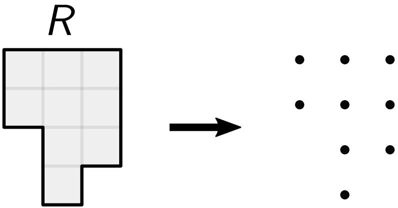

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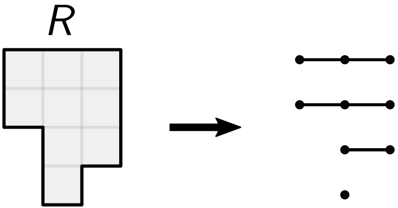

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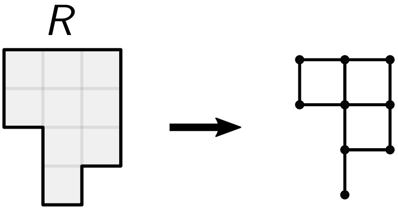
Forbidden Polyominoes

The 180-tromino tiling can also be reduced to the **Maximum Independent Set** problem.



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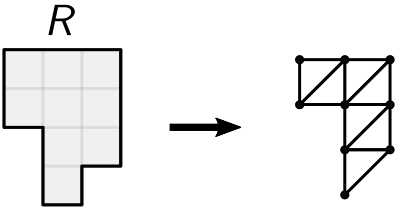

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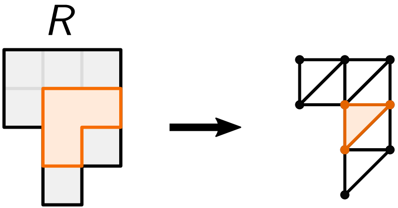

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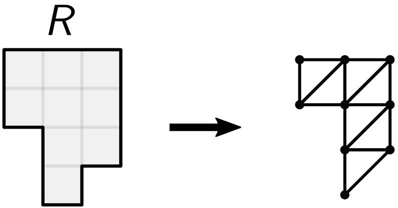

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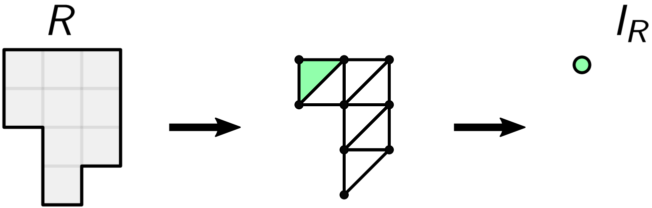

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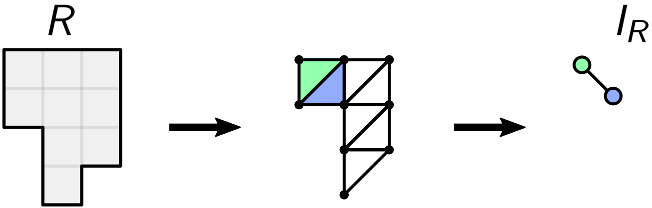

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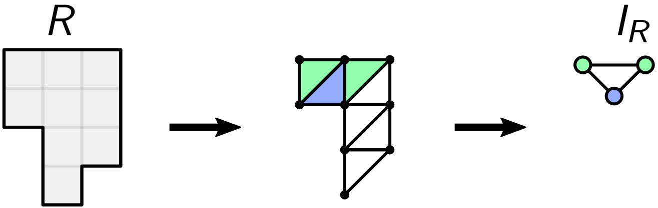

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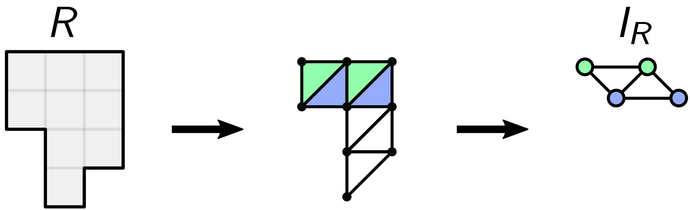

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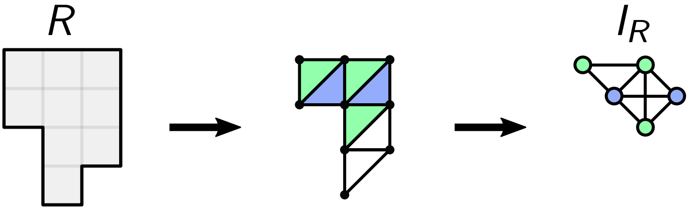

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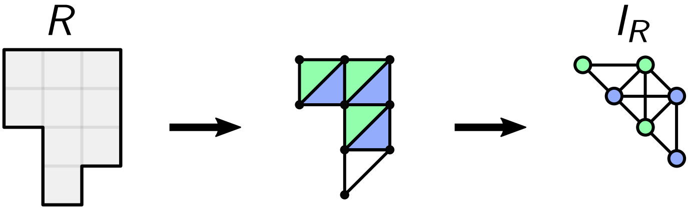

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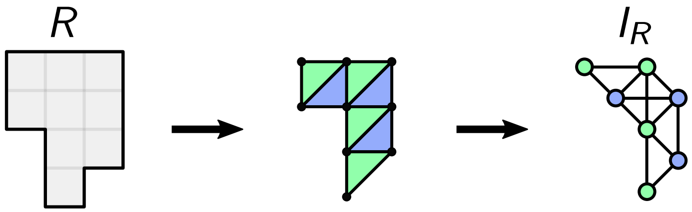

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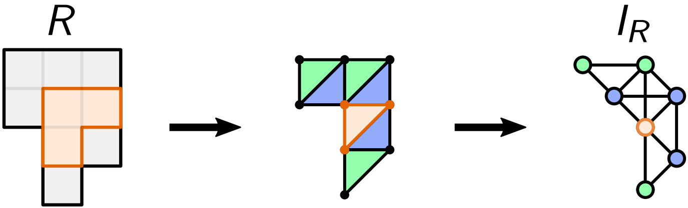

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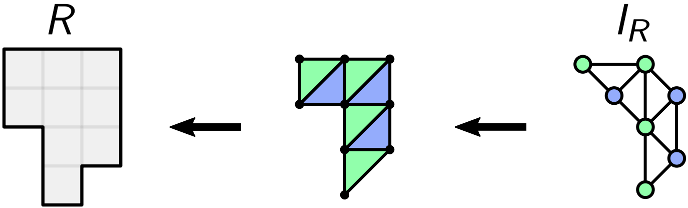

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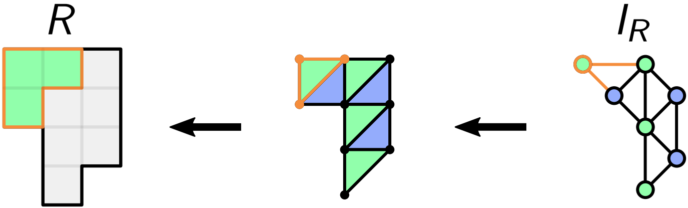

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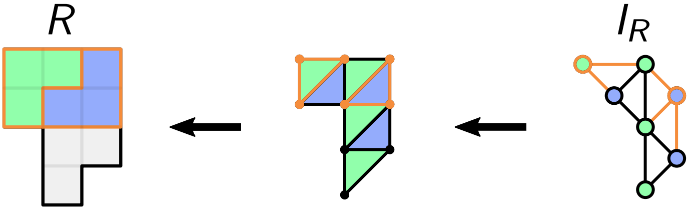

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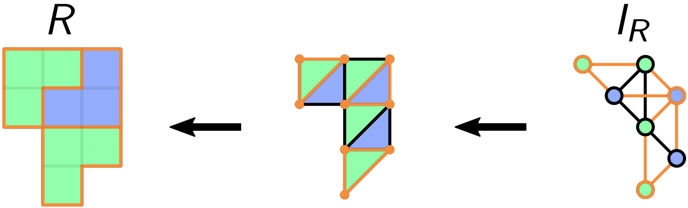

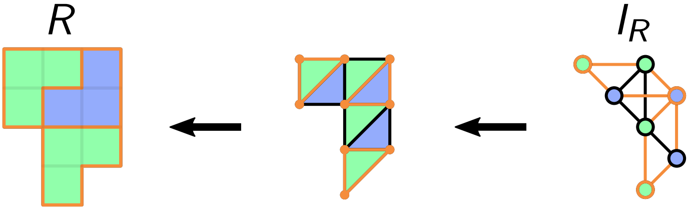

Theorem

Maximum Independent Set of I_R is equal to $\frac{|R|}{3}$
$\iff$ *R has a 180-tromino tiling .*

where $|R|$ the number of cells in a region R.

Forbidden Polyominoes (cont'd)

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If I_G is **claw-free**, i.e., does not contain a claw as induced graph, then computing Maximum Independent Set can be computed in polynomial time.

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The following five polyominoes generates a distinct I_G with a **claw** in it.

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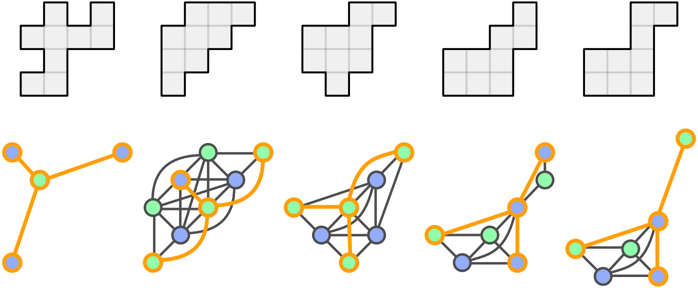

Forbidden Polyominoes (cont'd)

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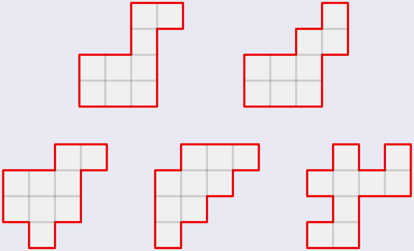
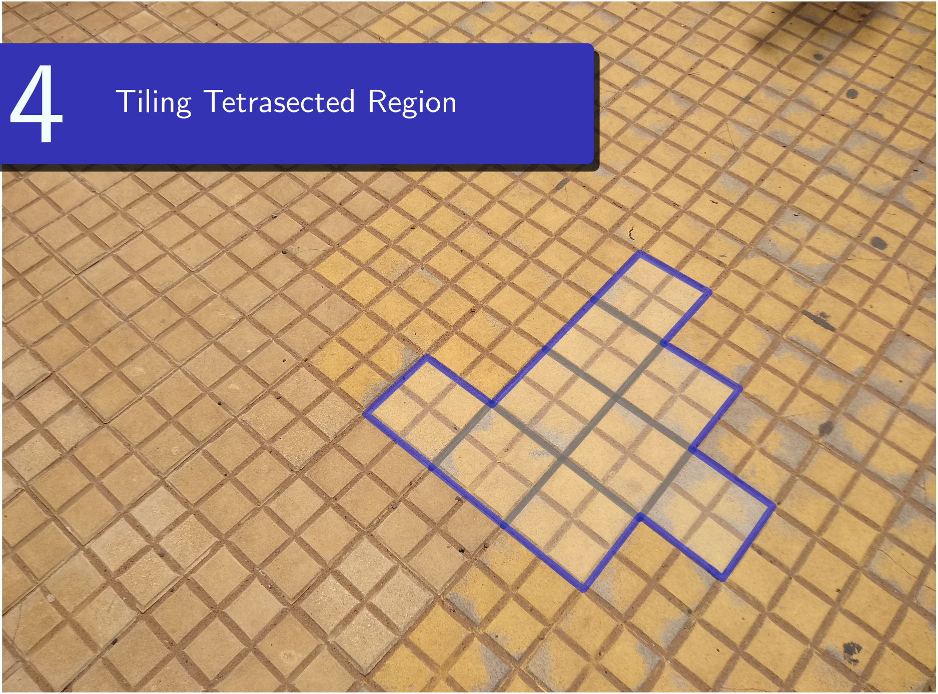
Theorem

*If a region R **doesn't** contains a rotated, reflected or sheared **forbidden polyomino**, then 180-tromino tiling can be computed in polynomial time.*



4 Tiling Tetrasected Region



Outline

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Tiling a Tetrasected Region

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Theorem

A tetrasected region $R^{\boxplus}$ has a L-tromino tiling if and only if $|R^{\boxplus}| \equiv 0 \pmod 3$. Furthermore, there exist a $O(n \log n)$ algorithm that computes the L-tromino tiling of $R^{\boxplus}$, where $n = |R|$.

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- **General case**.

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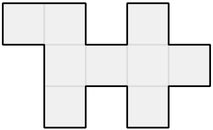

(a) A region R.

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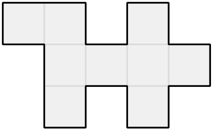

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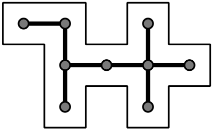

(b) Dual graph G_R.

Special case: the Dual Graph is a Tree (cont'd.)

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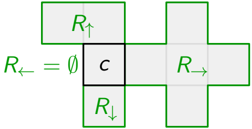

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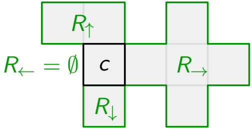

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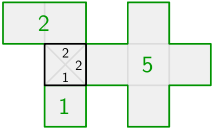

(b) Tags of c.

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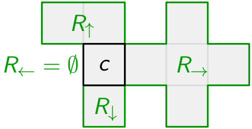

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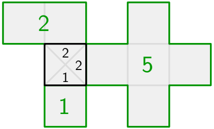

(b) Tags of c.

Lemma

The sum of the tags of any cell is always equal to $2 \pmod 3$.

Special case: the Dual Graph is a Tree (cont'd.)

There are exactly four kinds of **tagged cells**.

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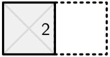

(a) Leaf

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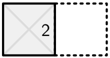

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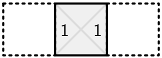

(b) Straight trunk

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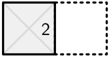

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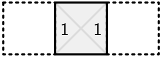

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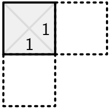

(c) Bent trunk

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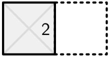

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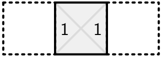

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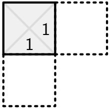

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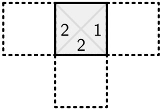

(d) 2-2-1 Fork

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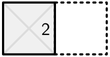

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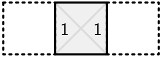

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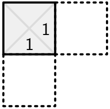

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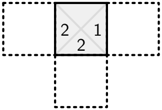

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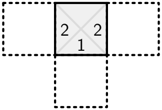

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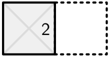

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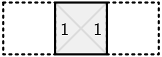

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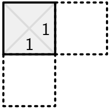

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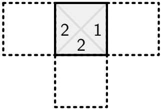

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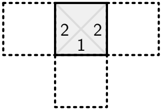

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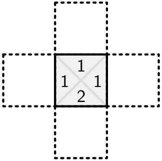
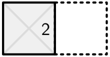
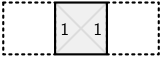


(f) 2^1 Cross

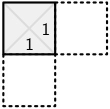
There are exactly four kinds of **tagged cells**.



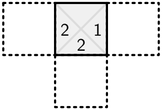
(a) Leaf



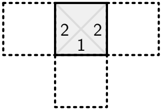
(b) Straight trunk



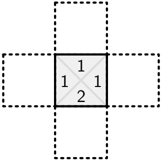
(c) Bent trunk



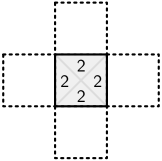
(d) 2-2-1 Fork



(e) 2-1-2 Fork



(f) 2^1 Cross



(g) 2^4 Cross

Caso especial cuando el Grafo Dual es un Árbol (cont.)

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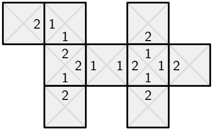

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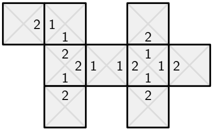

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If we replace each tagged cell with the following **tiling patterns**,

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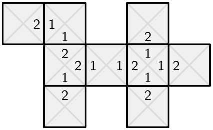

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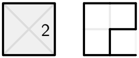

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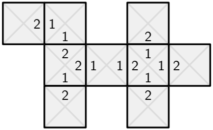

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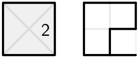

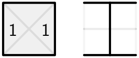

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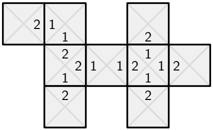

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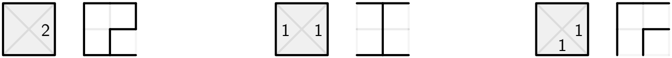

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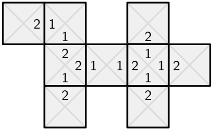

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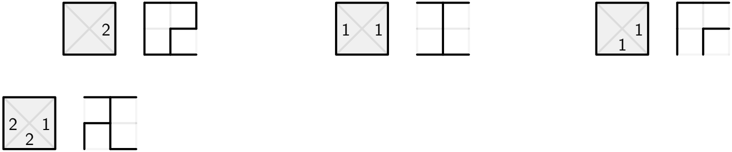

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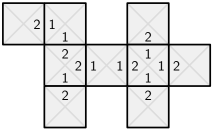

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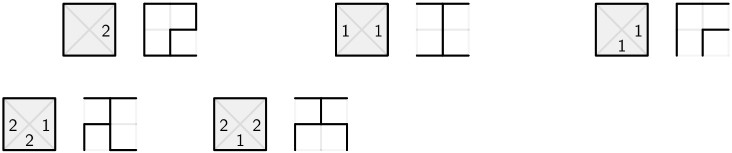

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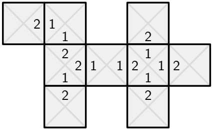

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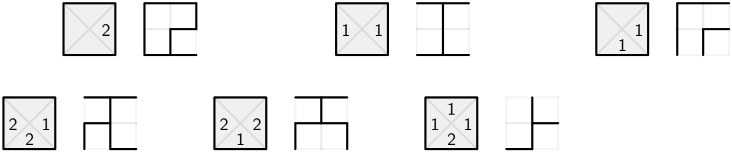

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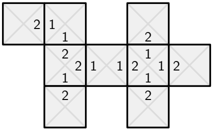

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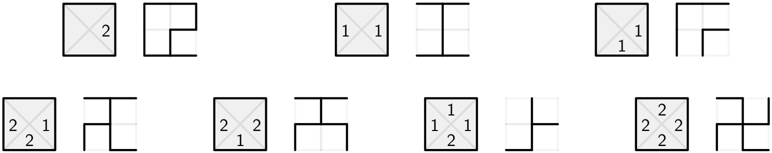

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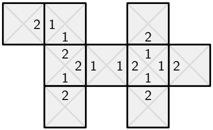

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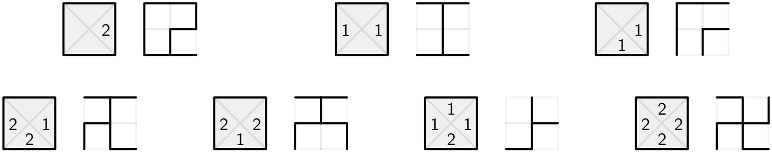

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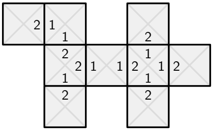

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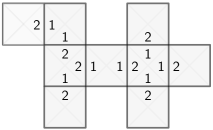

(b) After replacing with tiling patterns.

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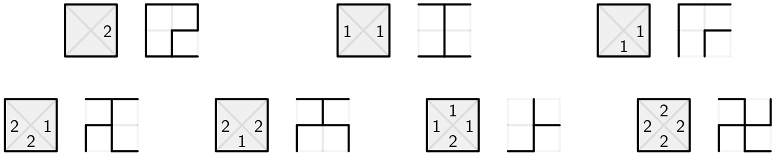

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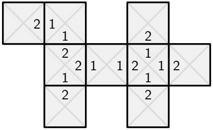

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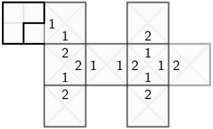

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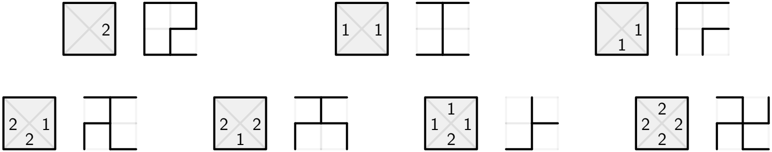

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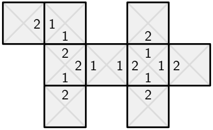

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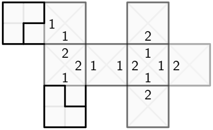

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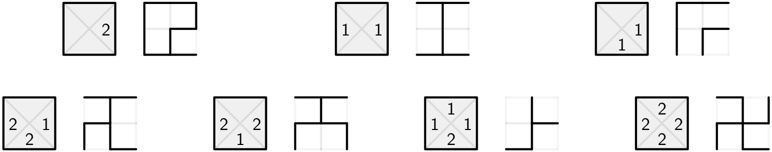

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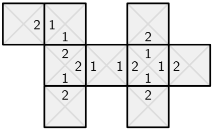

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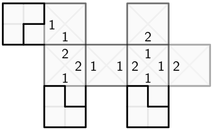

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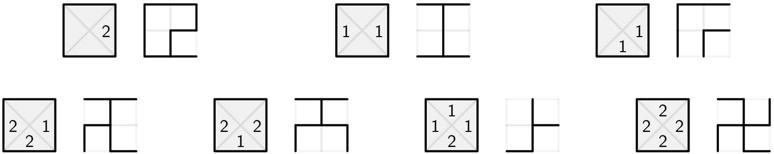

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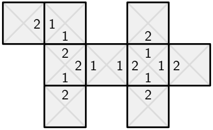

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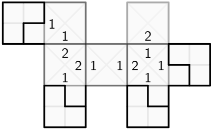

(b) After replacing with tiling patterns.

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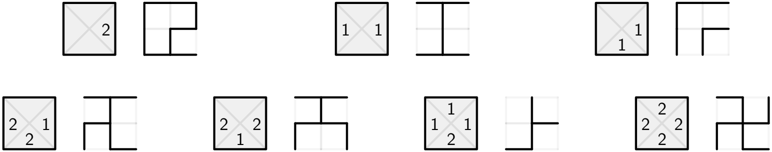

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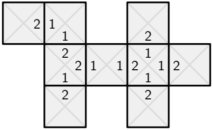

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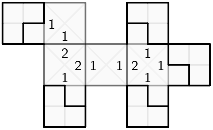

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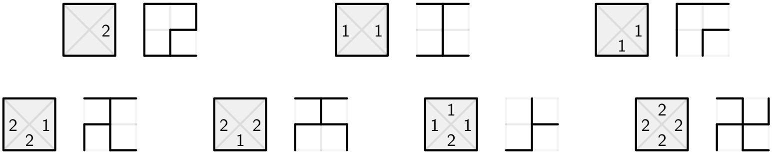

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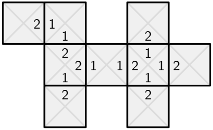

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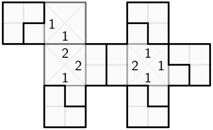

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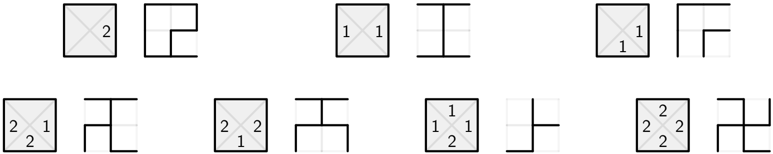

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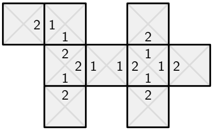

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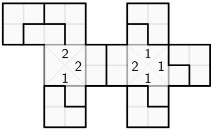

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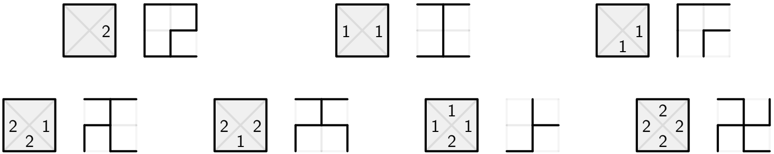

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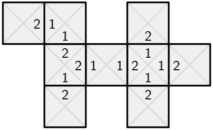

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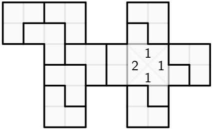

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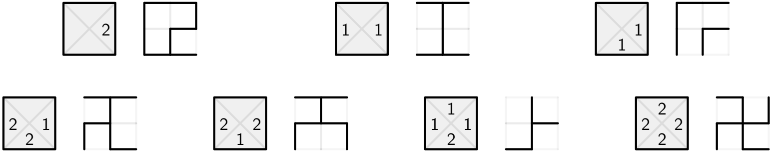

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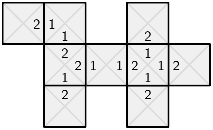

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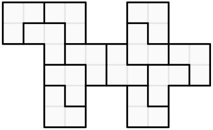

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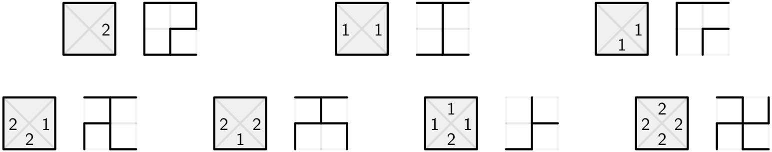

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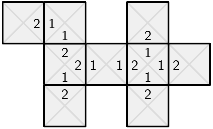

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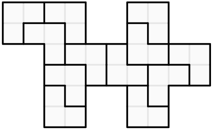

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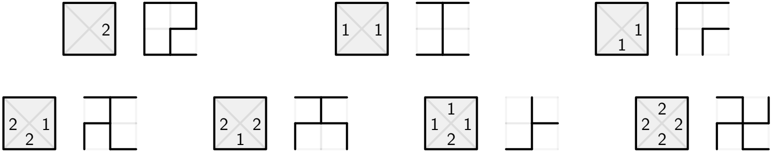

it will emerge a **L-tromino tiling** of the region $R^{\boxplus}$.

Outline

General case

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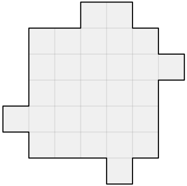

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In the **general case**, we consider a dual graph with **cycles**.

Given a region R such that $|R| \equiv 0 \pmod 3$, the following procedure will produce a L-tromino tiling of $R^{\boxplus}$:

- Partition the region R into two or more subregions such that:
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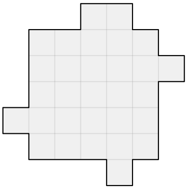

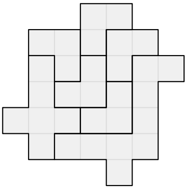

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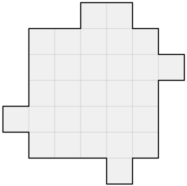

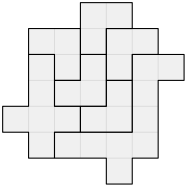

- Then, partition each subregion using the tiling pattern.

Caso general (cont.)

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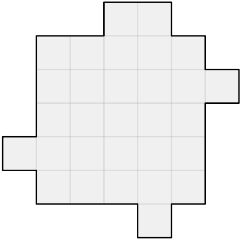

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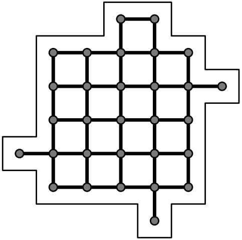

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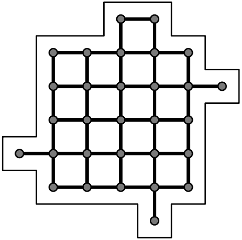

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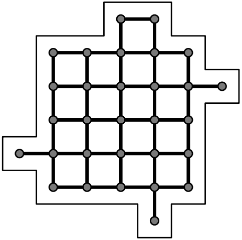

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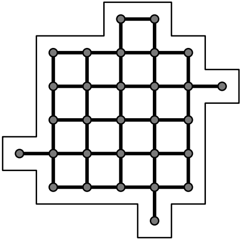

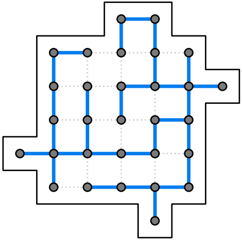

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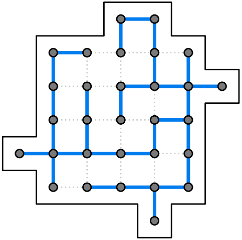

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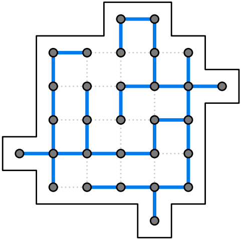

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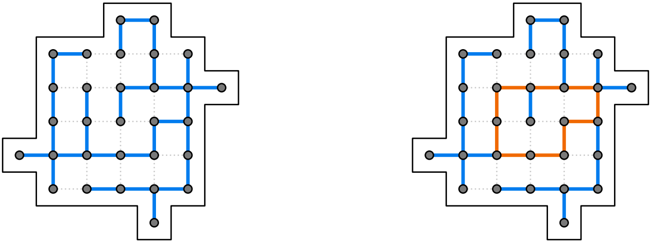

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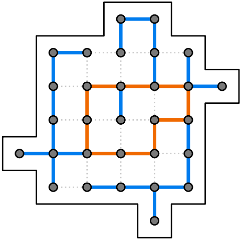

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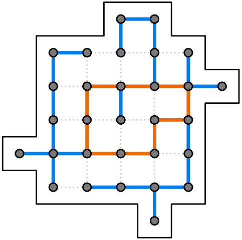

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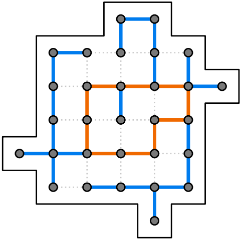

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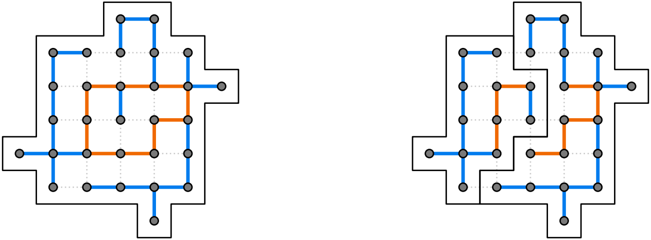

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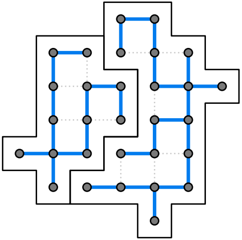

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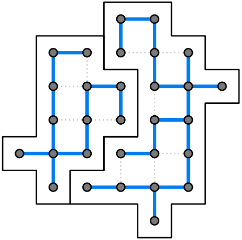

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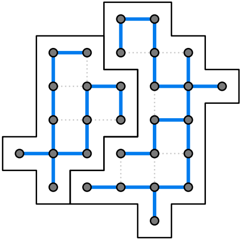

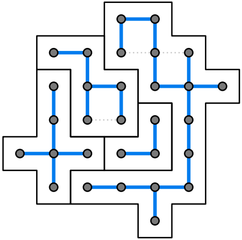

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