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What do we know about the product replacement algorithm? (English. English summary)

Groups and computation, III (*Columbus, OH*, 1999), 301–347, *Ohio State Univ. Math. Res. Inst. Publ.*, 8, *de Gruyter, Berlin*, 2001.

The product replacement algorithm is by far the most widely used algorithm for generating random elements in a finite group. The paper under review is a survey article which addresses theoretical aspects of the product replacement algorithm.

The algorithm consists in running a certain random walk on the set of k-tuples of group elements (see the paper [F. Celler et al., Comm. Algebra 23 (1995), no. 13, 4931–4948; MR 96h:20115] where the algorithm was introduced). The output after t steps is a random component of the resulting k-tuple. Different problems arise in the study of this random walk and of the dependence of its properties on the parameters k and t.

The first part of the paper is largely based on a joint paper of L. Babai and the author [in Proceedings of the Eleventh Annual ACM-SIAM Symposium on Discrete Algorithms (San Francisco, CA, 2000), 627–635, ACM, New York, 2000; MR 2001c:20157] and is about the bias in the output of the product replacement algorithm, which sometimes occurs if k is relatively small.

In the second part the author discusses the sparse available results on the connectivity of the underlying graph for small values of k, and gives an interesting survey of related problems in combinatorial group theory.

The third part of the paper is devoted to the quantitative bounds on the convergence of the random walk involved in the product replacement algorithm. The first such bounds were obtained by P. W. Diaconis and L. Saloff-Coste [Invent. Math. 134 (1998), no. 2, 251–299; MR 2000e:60013]. In many examples, however, the random walk mixes much more rapidly than these estimates indicate. Possible explanations of this phenomenon are discussed, based on a joint work of Alex Lubotzky and the author [J. Amer. Math. Soc. 14 (2001), no. 2, 347–363 (electronic)].

The paper concludes with a useful list of well-chosen references.

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