Partition bijections, a survey
Igor Pak
`https://www.math.ucla.edu/~pak/papers/psurvey.pdf`
preprint version of 2002-10-24
Errata and addenda by Darij Grinberg

1. Errata

"Line -k" means "line k from the bottom of the page".

- **page 6, line -3:** "between two set" $\rightarrow$ "between two sets".
- **page 7, second paragraph:** "its descriptions is" $\rightarrow$ "its description is".
- **page 7, third paragraph:** "we refer say" $\rightarrow$ "we say".
- **page 7, fourth paragraph:** "such that $\varphi \circ \beta = \alpha \circ \varphi\prime$" $\rightarrow$ "such that $\beta \circ \varphi = \varphi\prime \circ \alpha$".
- **page 7, sixth paragraph ("The notation"):** What do you mean by "both notations" in "We use routinely both notations"?
- **page 8, §2.1.1:** In the first sentence, replace "λ_l" by "λ_ℓ" ("\ell", not just "l") in the first chain of inequalities.
- **page 8, §2.1.1:** At the end of the first paragraph of §2.1.1, replace "$|\ell(\lambda')|$" by "$\ell(\lambda')$". (I am not saying it's wrong with $|\ell(\lambda')|$ ...)
- **page 9, §2.1.4:** "$[\mu]_2$ to be a Young diagram" $\rightarrow$ "$[\mu]_2$ is defined to be a Young diagram".
- **page 9, §2.1.4:** "no 2 can appear above 1" $\rightarrow$ "no 2 can appear below a 1".
- **page 9, §2.1.4:** In the definition of an m-modular diagram $[\mu]_m$, you seem to have fogrotten the requirement that the numbers should decrease down each column.
- **page 11, §2.2.3:** In the rightmost handmost side of the second displayed equation, replace "$s^n P_k(t)$" by "$s^n(P_n(t) - P_{n-1}(t))$".
- **page 11, §2.3.1:** In the first sentence, I would replace "$1 \leq i, j \leq r$" by "$1 \leq i \leq r$ and $1 \leq j \leq r$". (The comma might otherwise be misread as separating two inequalities, i.e., as "$1 \leq i$ and $j \leq r$".)
- **page 11, §2.3.1:** "such that $\mu, \nu' \in \mathcal{P}_{n,k}$" $\rightarrow$ "such that μ, ν' have length at most r each".

- **page 11, §2.3.1:** "by letting $\varphi(\lambda) = (\mu, \nu)$" $\to$ "by letting $\varphi(\lambda) = (\mu, \nu')$".
- **page 11, §2.3.1:** "*Cauchy's idenity*" $\to$ "*Cauchy's identity*".
- **page 12, §2.3.4:** "for a partition" $\to$ "form a partition".
- **page 12, §2.3.5:** "ont the bottom" $\to$ "on the bottom".
- **page 13, §2.3.6:** The sum on the left hand side of Ramanujan's identity should start at $m = 1$, not at $m = 0$ (otherwise, the constant terms don't match).
- **page 13, §2.3.6:** "unique smallest part $s(\mu)$" $\to$ "unique smallest part $s(\lambda)$".
- **page 14, §2.4.2:** "or $(i-1, j)$m" $\to$ "or $(i-1, j)$,".
- **page 14, §2.4.2:** The definition of the outside boundary is wrong for $\lambda = \varnothing$ (here, no square fits your description; however, the cell $(1, 1)$ should belong to the outside boundary $[\varnothing]$).
- **page 16, §2.5.4:** "$\sum_{n=1}^{\infty} h(n, -r)\, t^r$" $\to$ "$\sum_{n=1}^{\infty} h(n, -r)\, t^n$" in the first sentence.
- **page 16, §2.5.7:** "$vr(\lambda, \mu, \nu)$" $\to$ "$\rho(\lambda, \mu, \nu)$".
- **page 19, §2.7.6:** The "n"s on the left hand side should be "k"s. Also, the products on the left hand side should be correctly interpreted for negative k; they aren't just empty products in this case.
- **page 20, §3.2.4:** "into an even and an odd number of even parts" $\to$ "with an even and an odd number of even parts" (the partitions can have odd parts, too).
- **page 23, §4.1:** The sum on the left hand side of Lebesgue's identity should start at $r = 0$, not at $r = 1$.
- **page 27, §4.4.2:** The identity has a lot of typos; its correct version is

$$\sum_{n=0}^{\infty} q^{n^2} \frac{(1+q)\left(1+q^3\right)\cdots\left(1+q^{2n-1}\right)}{\left(1-q^2\right)^2\left(1-q^4\right)^2\cdots\left(1-q^{2n}\right)^2} = \prod_{n=1}^{\infty} \frac{1+q^{2n-1}}{1-q^{2n}}.$$

- **page 29, §5.1.1:** Replace "$(-1)^{m-1}$" by "$(-1)^m$".
- **page 30, §5.1.2:** "$+(-1)^m \left(n - \frac{m(3m-1)}{2}\right) + \left(n - \frac{m(3m+1)}{2}\right)$" should be "$+(-1)^{m-1} p\left(n - \frac{m(3m-1)}{2}\right) + (-1)^{m-1} p\left(n - \frac{m(3m+1)}{2}\right)$".

- **page 32, §5.3.1:** In the first display here, replace "$1 - xt^n$" by "$1 - xt^i$".
- **page 33, §5.4.1:** "how where" $\to$ "where".
- **page 34, §5.5.4:** The product sign on the left hand side should have an "$m = 1$" instead of an "$n = 0$" underneath it.
- **page 35, §6.1.4:** The display should end with a period, not a comma.
- **page 35, §6.1.5:** Remove the comma before "such that".
- **page 37, §6.3:** "$(-1)^n$" should be "$(-1)^k$" on the right hand side.
- **page 37, §6.3:** Replace "$q^2 = uv,\ z = -u/v$" by "$u = -q/z$ and $v = -qz$".
- **page 45, §8.4.2:** "give a perfect matching on a set $F = [N]^{D_\infty}$ of fixed points of the action of D_∞ on $[N]$" $\to$ "give a perfect matching on the set of points $i \in [N]$ that are fixed by exactly one of α and β".

  (The points that are fixed under both α and β are unmatched, and their number can have any parity.)

  Incidentally, I am not sure how useful it is to frame this all in terms of a group action instead of just using two involutions α and β of a finite set (not necessarily $[N]$).
- **page 51, §9.2.1:** "translates into $|\mu_i - \nu_i| \leq 1$" $\to$ "translates into $\left|\mu'_i - \nu'_i\right| \leq 1$".
- **page 51, §9.2.2:** Where does the ℓ appear on the right hand side?
- **page 53, §9.4.1:** It should be explained that you define $\zeta(n) = 0$ for all $n \leq 0$ (as these kinds of terms can appear in the formula).
- **page 53, §9.4.1, proof:** "where $\mathcal{D}$ is a set" $\to$ "where $\mathcal{D}$ is the set".
- **page 53, §9.4.1, proof:** "from the proof of 5.1.1": Which of the proofs?
- **page 53, §9.4.1, proof:** In the long computation, the sum over "$\lambda \in \mathcal{O}_{n-m}$" should perhaps be a sum over "$\lambda \in \mathcal{D}_{n-m}$" instead?
- **page 53, §9.4.1, proof:** "where $m_d(\lambda)$ is a multiplicity" $\to$ "where $m_d(\lambda)$ is the multiplicity".
- **page 53, §9.4.1, proof:** After "Now, adding part d to λ", add "and decrementing c by 1".
- **page 55, §9.5.6:** "and $w, x, z, u \geq 0,\ y > 0$" $\to$ "and $w, x, z, u > 0$ and $y \geq 0$" (the two inequality signs need to trade places).
- **page 59, item 3:** "Euler's Theorem **??**" is a missing reference.