

## your thm?

Igor Pak <pak@math.ucla.edu> Reply-To: pak@math.ucla.edu To: John Stembridge <jrs@umich.edu> Wed, Feb 17, 2021 at 6:06 AM

Dear John,

In a paper I am currently writing I am citing the following result as your theorem. Is that the case? What's the story here? This is a snippet from "Coincidences among skew Schur functions" by V Reiner, KM Shaw, and S Van Willigenburg. Thanks! Best, -- Igor

We close this section with an interesting special case of Corollary 7.31, which was first pointed out to us by John Stembridge and for which we offer two proofs.

**Corollary 7.32.** For any Ferrers diagram  $\mu$  contained in the staircase partition  $\delta_n := (n - 1, n - 2, ..., 1) \vdash {n \choose 2}$ , one has

 $\delta_n/\mu \sim \left(\delta_n/\mu\right)^t$ .



John Stembridge <jrs@umich.edu> To: pak@math.ucla.edu Cc: John Stembridge <jrs@umich.edu>

Dear Igor,

Yes, that result is due to me, but I never published it. (Perhaps others discovered it independently?) I had to look in some very old emails to recall the context, but apparently it happened at a 2004 CMS conference where Stephanie van Willigenburg gave a talk about her paper with Lou Billera (et.al.?) on equalities between ribbon Schur functions.

Here is a clip from a 2006 email of mine to Vic Reiner when they were revising their equality-of-Schurs paper:

> My memory, which could of course be faulty, is that at the end of

> her talk on the ribbons paper, I asked her if she had thought about the

> equal Schur problem, and her answer was negative. I had my laptop there,

> and there was free time after her talk.... After about 15 minutes,

> I was generating examples, and noticed immediately that the smallest

> example was not a ribbon. Several people took an interest in what I was

> doing, including as I remember Hugh Thomas, Peter McN, and Steph.

> I think Francois was there, but I don't recall him taking an interest.> By the next day, I had proved "Cor 7.31", and I remember distinctly

> sitting at the conference banquet with Richard and Steph,

> mentioning Cor 7.31, and talking over how the equal Schur question

> looked quite promising. After the conference, I dabbled around with

> the problem (e.g., I had a conjectured solution for all 4-rowed

> equalities, and some general ideas for attacking the problem),

> but eventually dropped it. I had been planning to hand it to a grad

> student, but she (temporarily) dropped out of our program, so I put

> it on a metaphorical shelf.

BTW: the grad student who dropped out re-entered our program a few years later. I gave her a related thesis topic in which she investigated equalities between (normalized) Schur Q-functions and also equalities between Schur Q's and Schur s's. I mention this because the (staircase)/mu skew shapes play a prominent role in her 2012 thesis. It can be downloaded here:

## https://deepblue.lib.umich.edu/handle/2027.42/93841

Best, John

> ------ you wrote -----Date: Wed, 17 Feb 2021 17:06:31 +0300 To: John Stembridge <jrs@umich.edu> From: Igor Pak <pak@math.ucla.edu>

Subject: your thm? [Quoted text hidden] Igor Pak <igor.pak.la@gmail.com>

Wed, Feb 17, 2021 at 7:32 AM