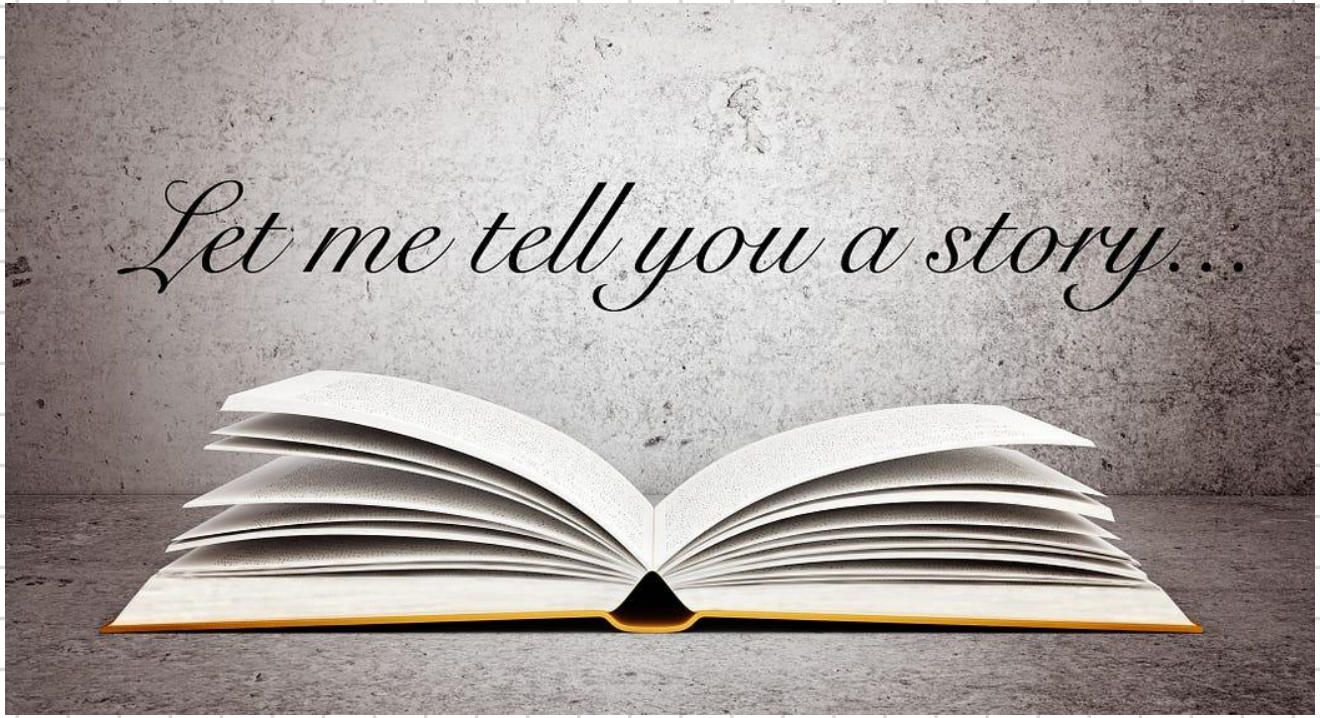
Igor Pak, UCLA

Dec 10, 2025

The story of the Bunkbed Conjecture

IPAM Reunion Workshop, UCLA, Lake Arrowhead, CA

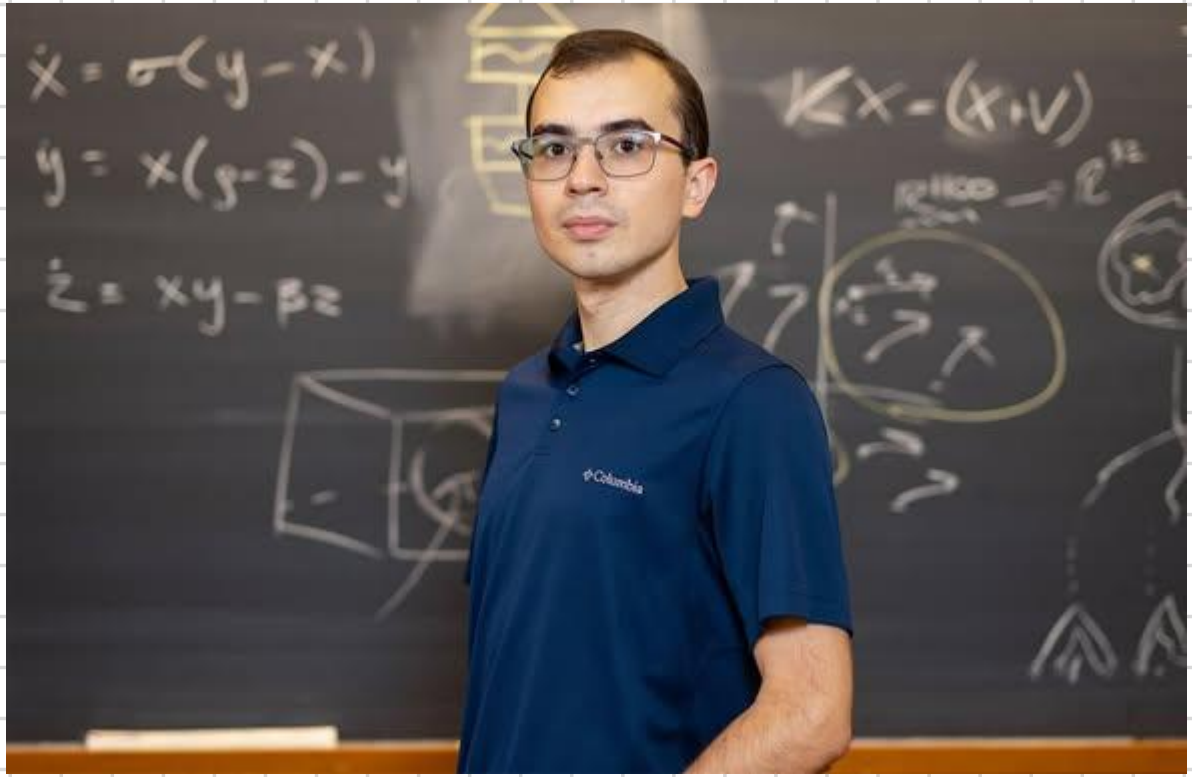
Warning: This is a story, not a technical talk



Main characters

Nikita Gladkov,
University of Geneva
(UCLA PhD’25)

Main characters



Alexander (Sasha) Zimin,
MIT (PhD’26, exp.)

Bunkbed Conjecture

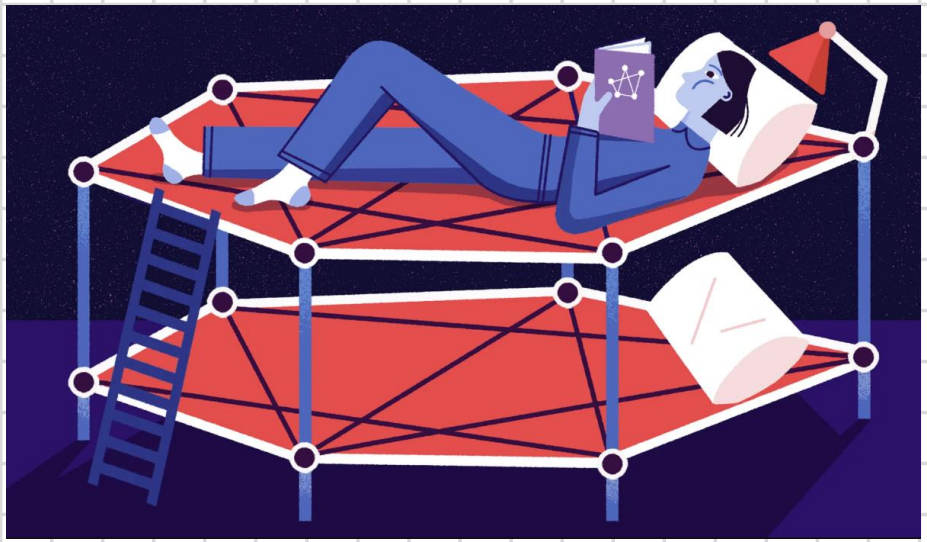
Definition (*bunkbed graph*)

$G = (V, E)$ $\longleftarrow$ simple graph

$G' = (V', E')$ $\longleftarrow$ exact copy of G

$S \subseteq V$ $\longleftarrow$ set of pole vertices

Γ $\longleftarrow$ union of G, G' and *pole edges* (v, v'), $v \in S$



Bunkbed Conjecture (Kasteleyn, c. 1985)

Take bond p-percolation on Γ *where pole edges are always open.*

Then: $\mathbb{P}_p[u \leftrightarrow v] \geq \mathbb{P}_p[u \leftrightarrow v']$ *for all* $u, v \in V$.

Main result

Theorem (Gladkov–P.–Zimin'25)

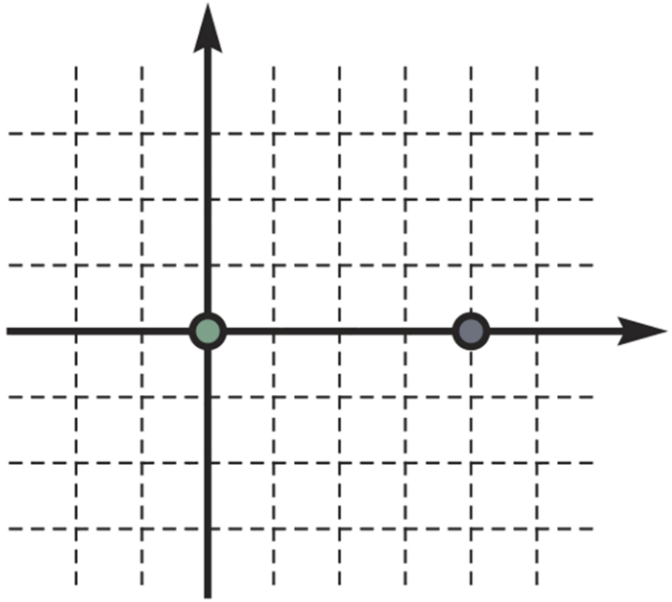
The Bunkbed Conjecture is false.

PNAS RESEARCH ARTICLE | MATHEMATICS

The bunkbed conjecture is false

Nikita Gladkov[a,1], Igor Pak[a,1,2], and Aleksandr Zimin[b,1]

Brief history



Monotonicity Conjecture: (Hammersley–Welsh'65)

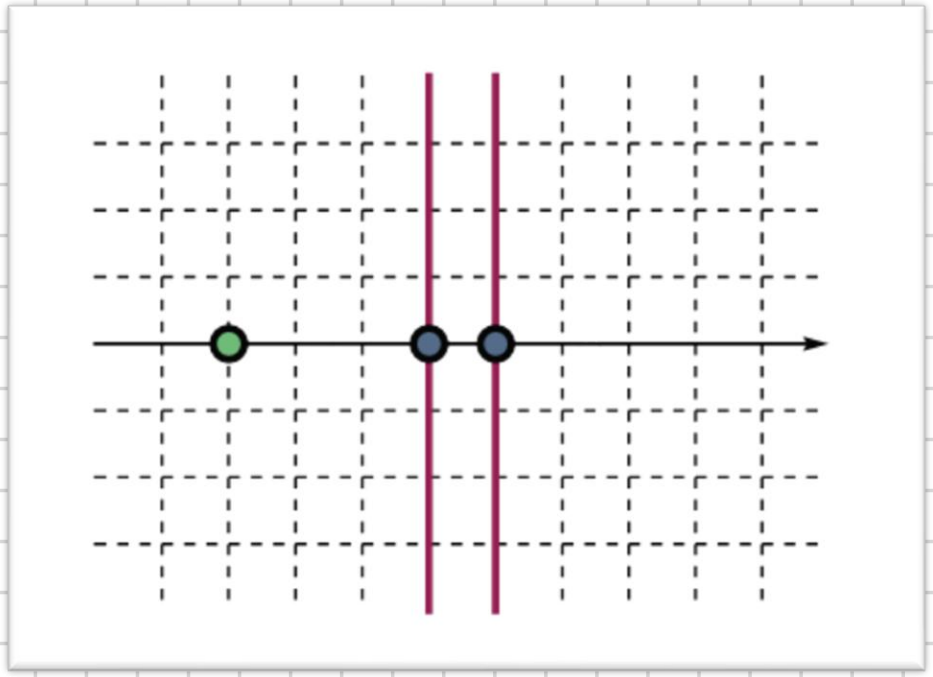
Take bond p-percolation on $\mathbb{Z}^2$. *Then :*

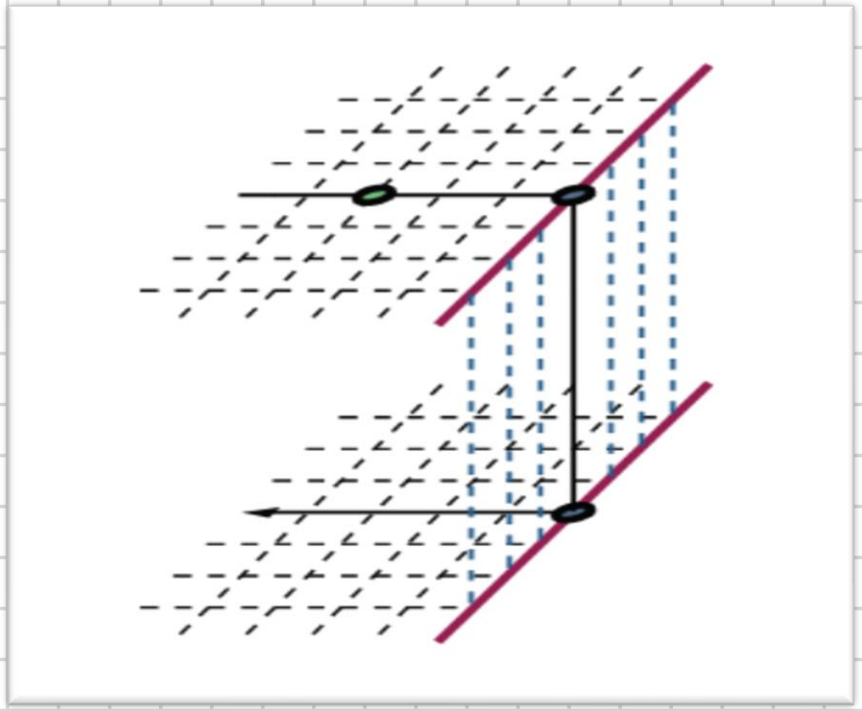
$$\mathbb{P}_p\big[(0,0) \longleftrightarrow (n,0)\big] \;\geq\; \mathbb{P}_p\big[(0,0) \longleftrightarrow (n+1,0)\big].$$

Brief history

Kasteleyn's Observation:

Bunkbed Conjecture $\Longrightarrow$ *Monotonicity Conjecture*





Brief history

BBC holds for:

- wheel graphs (Leander'09)
- complete graphs (de Buyer'18, van Hintum-Lammers'19)
- graphs with $u \leftrightarrow v$ symmetry (Richthammer'22)
- trees (Donderwinkel-Jorritsma-Perarnau'25)
- theta graphs (Lněničková'25)
- cactus graphs (= every two cycles intersect, Denart'25)
- $|S|=1$ (folklore, Linusson'11)
- $|S|=2$ (van den Berg-Kahn'01 + Gladkov'25)
- $p \uparrow 1$ (Hutchcroft-Kent-Nizic'23)

Note: $p \uparrow 1$ case of the *Monotonicity Conjecture* proved in [de Lima-Procacci-Sanchis'15]

Brief history

Theorem (formerly *Bollobás–Brightwell Conjecture*, Häggström'98)

BBC type inequality holds for continuous time random walks on bunkbed graphs.

Note: Proof by a coupling argument.

Theorem (Häggström'03)

BBC type inequality holds for the Ising model on bunkbed graphs.

Note: Proof via new FKG type correlation inequality.

Personal history

Tom Liggett (c. 2009):

“Igor, this is a kind of problem you might like, take a look!”

Personal history

Chapter 42
Selected Problems in Probability Theory

Geoffrey R. Grimmett

42.4.1 Bunkbed Inequality

The mysterious 'bunkbed' inequality was posed by Kasteleyn around 1985

Our paper

arXiv > math > arXiv:2410.02545

Mathematics > Combinatorics

[Submitted on 3 Oct 2024]

The bunkbed conjecture is false

Nikita Gladkov, Igor Pak, Aleksandr Zimin

Igor Pak's blog

Views on life and math

The bunkbed conjecture is false

October 1, 2024 igorpak

What happened

All-time stats

The bunkbed conjecture is false

Published Oct 1, 2024

Views

40.8K

r/math • 1y ago
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The bunkbed coniecture is false

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The bunkbed conjecture is false (igorpak.wordpress.com)

189 points by surprisetalk on Oct 2, 2024 | hide | past | favorite | 71 comments

What happened

All-time stats

The bunkbed conjecture is false

Published Oct 1, 2024

Views

40.8K

What happened

XJMR

HOME » MATH » MATH LOUNGE (OFF TOPIC)

Bunkbed conjecture is False

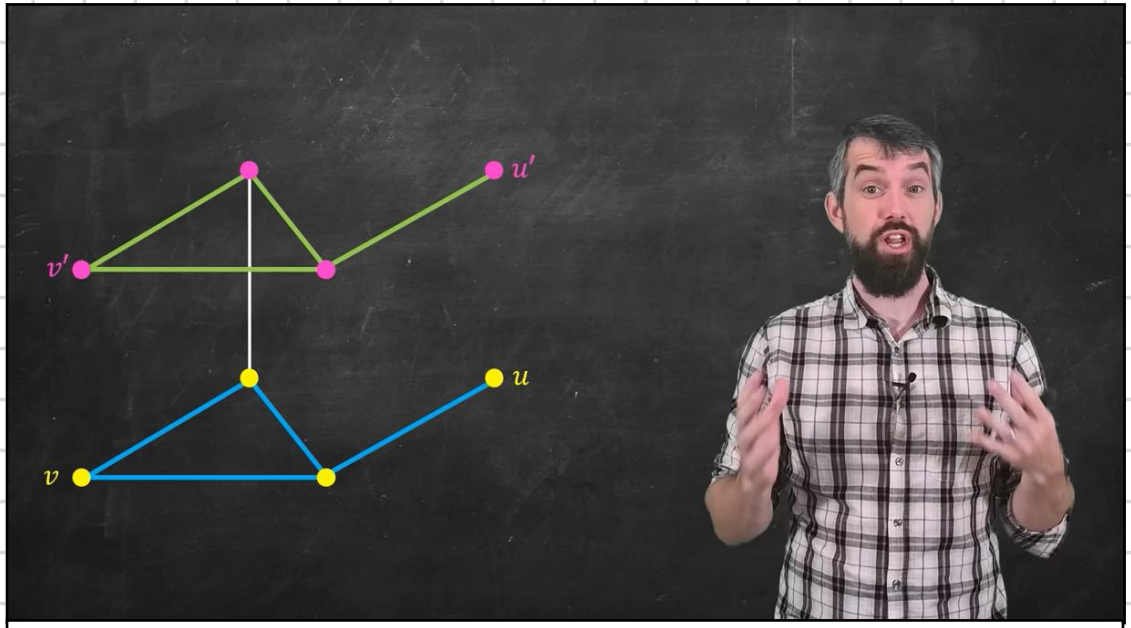
Economist
Vader

This post makes me think less of Igor Pak (not that I ever thought much)

Economist
Miller

No one cares about "bunkbed" conjecture.

What happened



Math News: The Bunkbed conjecture was just debunked!!!!!!!

Dr. Trefor Bazett ✓
561K subscribers
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11K

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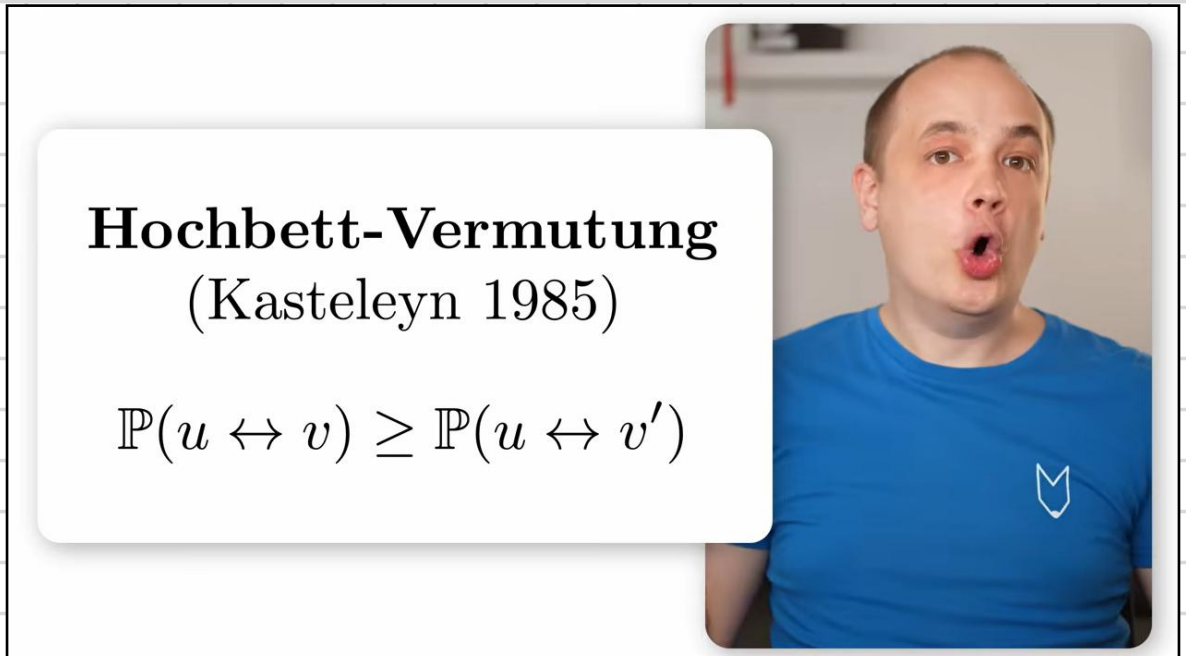

Mathe-News: 🚨 Die Hochbett-Vermutung ist falsch!

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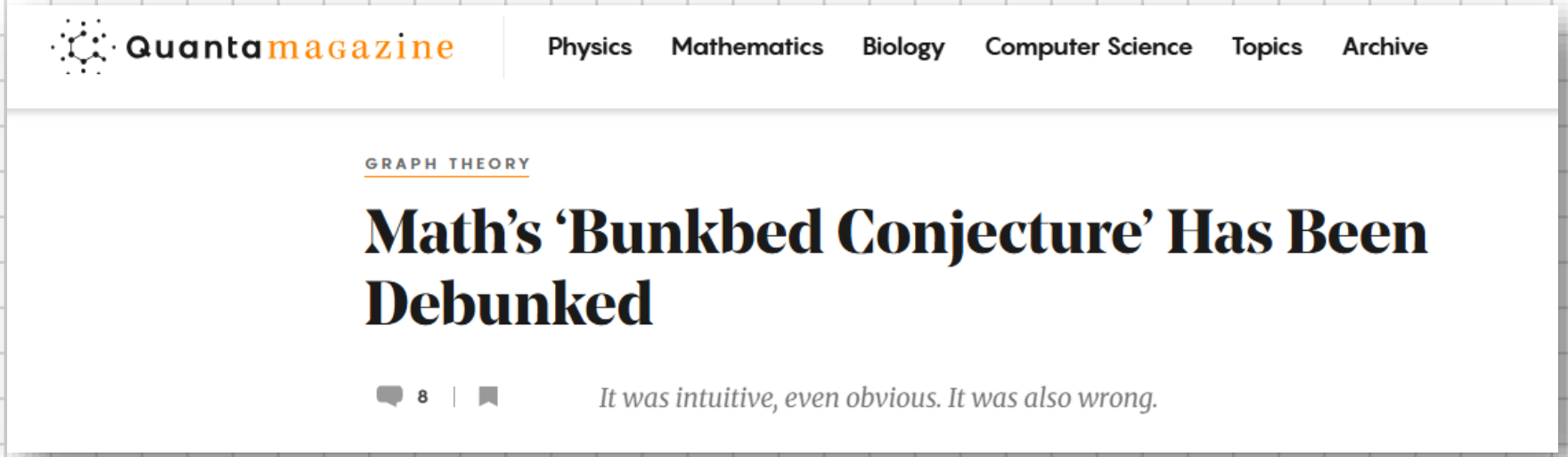
What happened



What happened

Pour la Science

Mathématiques

Théorie des graphes : la conjecture du lit superposé est fausse

Frankfurter Allgemeine

Wenn Mathematiker im Stockbett liegen

epsilon
le magazine scientifique d'actualité

Les livres

"Beaucoup de conjectures mathématiques sont fausses"

搜狐 SOHU.com

UCLA、MIT数学家推翻39年经典数学猜想！AI证明卡在99.99%，人类最终证伪

2024-11-04 13:06　来源: 新智元　发布于：北京市

What happened

SecurityLab.ru
by Positive Technologies

МЕНЮ

Главная / Новости /

Путь наверх оказался короче: математики опровергли «Гипотезу о двухъярусной кровати»

НЕЗАВИСИМАЯ

Газета Наука и технологии Печатная версия

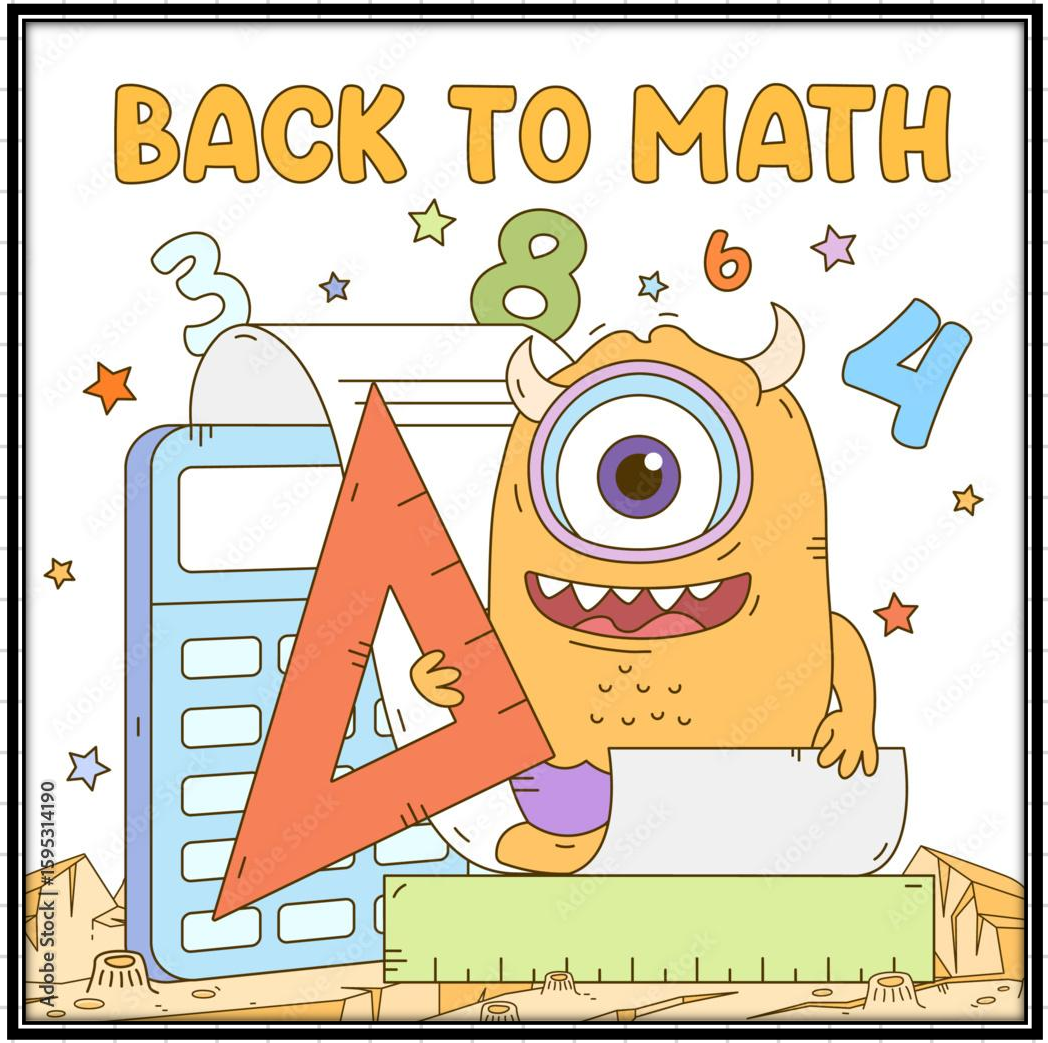
21.01.2025 18:00:00

Новости науки. Российские учение опровергли известную в математике гипотезу о двухъярусной кровати

ТАСС НАУКА

13 ЯНВ, 00:32

Математики РФ и США опровергли гипотезу, считавшуюся верной на протяжении 40 лет


BACK TO MATH
3
8
6
4
Adobe Stock | #595314190

How we started

Motivation: Combinatorial interpretation for *Kronecker coefficients*

Question 1: Is the defect of the (scaled) bunkbed inequality in #P?

$p = \frac{1}{2}$

Explanation*:* Both the coupling and the FKG define explicit (recursively defined) injections. Thus a negative answer to **Question 1** would rule out these types of arguments.

How we started

Polynomial BBC

$$\boldsymbol{x}^{C \sqcup C'} := \prod_{e \in C} x_e \prod_{e' \in C'} x_e$$

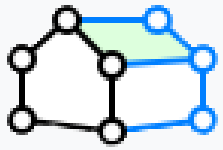
$$\mathcal{F}_0 := \sum_{\left\{C \subseteq 2^E, C' \subseteq 2^{E'} \,:\, u \leftrightarrow v\right\}} \boldsymbol{x}^{C \sqcup C'}, \qquad \mathcal{F}_1 := \sum_{\left\{C \subseteq 2^E, C' \subseteq 2^{E'} \,:\, u \leftrightarrow v'\right\}} \boldsymbol{x}^{C \sqcup C'}$$

Question 2: Is $\mathcal{F}_0 \geqslant \mathcal{F}_1$ term-wise?

$$x^2 + y^2 \geqslant 2xy$$

Note: BBC $\implies \mathcal{F}_0(\boldsymbol{x}) \geq \mathcal{F}_1(\boldsymbol{x})$ for all $\boldsymbol{x} \in [0,1]^{|E|}$

How we started



The House of Graphs

Proposition:

Polynomial BBC holds for all graphs with $|V| \leq 8$

and for all graphs with $|E| \leq 15$.

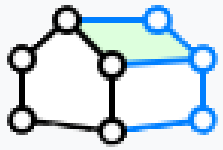
First part: 12,114 graphs

A001349	Number of simple connected graphs on n unlabeled nodes. (Formerly M1657 N0649)

1, 1, 1, 2, 6, 21, 112, 853, 11117, 261080, 11716571, 1006700565

OEIS

How we started



The House of Graphs

Proposition:

Polynomial BBC holds for all graphs with $|V| \leq 8$ and for all graphs with $|E| \leq 15$.

Second part: 2,471,798 graphs

A046751

```
1
0 1 1
0 0 2 2 1 1
0 0 0 3 5 5 4 2 1 1
0 0 0 0 6 13 19 22 20 14 9 5 2 1 1
0 0 0 0 0 11 33 67 107 132 138 126 95 64 40
0 0 0 0 0 0 23 89 236 486 814 1169 1454 1579 1515
0 0 0 0 0 0 0 47 240 797 2075 4495 8404 13855 20303
0 0 0 0 0 0 0 0 106 657 2678 8548 22950 53863 112618
0 0 0 0 0 0 0 0 0 235 1806 8833 33851 109844 313670
0 0 0 0 0 0 0 0 0 0 551 5026 28908 130365 499888
0 0 0 0 0 0 0 0 0 0 0 1301 13999 93569 489387
0 0 0 0 0 0 0 0 0 0 0 0 3159 39260 300748
0 0 0 0 0 0 0 0 0 0 0 0 0 7741 110381
0 0 0 0 0 0 0 0 0 0 0 0 0 0 19320
```

How we failed

Experimental Conclusion:
BBC and Polynomial BBC either both true or both false. Thus one should aim to disprove *the BBC.*

Mathematics > Combinatorics

arXiv:2104.14516 (math)

[Submitted on 29 Apr 2021]

Constructions in combinatorics via neural networks

Adam Zsolt Wagner

AI testing

We ran Wagner's 2021 algorithm applied to Linusson's Alternative BBC (equivalent to BBC). We implemented it in Julia to test the conjecture on graphs with n = 20 and 30 vertices and 3,4,5 pole vertices. We run the code on *UCLA Hoffman2 Linux Cluster* (800+ 64-bit nodes, over 26,000 cores, over 174 TB of memory). At every step we took N = 4,000 random samples. Each run took about 2 hours.

The resulting graphs were relatively dense (around 100 edges) and both BBC probabilities within 0.01, without producing a counterexample.

How we failed

AI testing (continued)

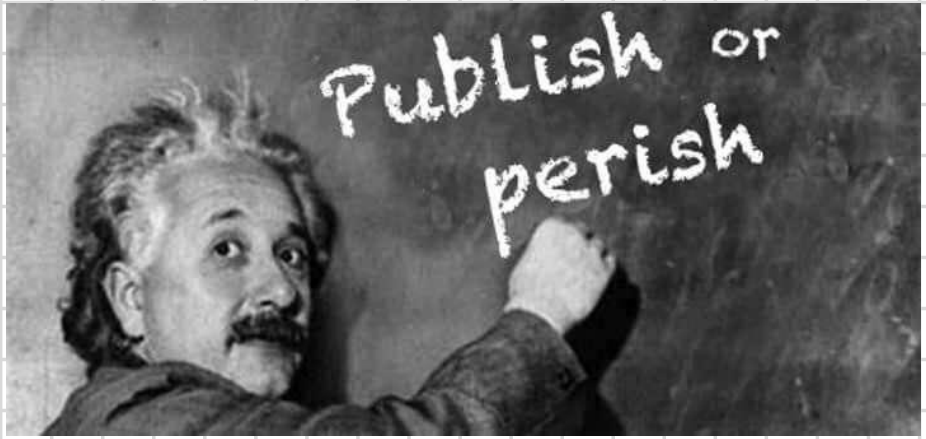
The resulting graphs were relatively dense (around 100 edges) and both BBC probabilities within 0.01, without producing a counterexample.

It became clear that one would need to increase *N* significantly to continue testing, and that the resulting potential counterexample would be out of reach by an *exact computation*.

How we failed

Observations:

There is no ***5 sigma*** standard in math. If you want to publish a counterexample in a math journal you need a formal proof, not a probabilistic proof.

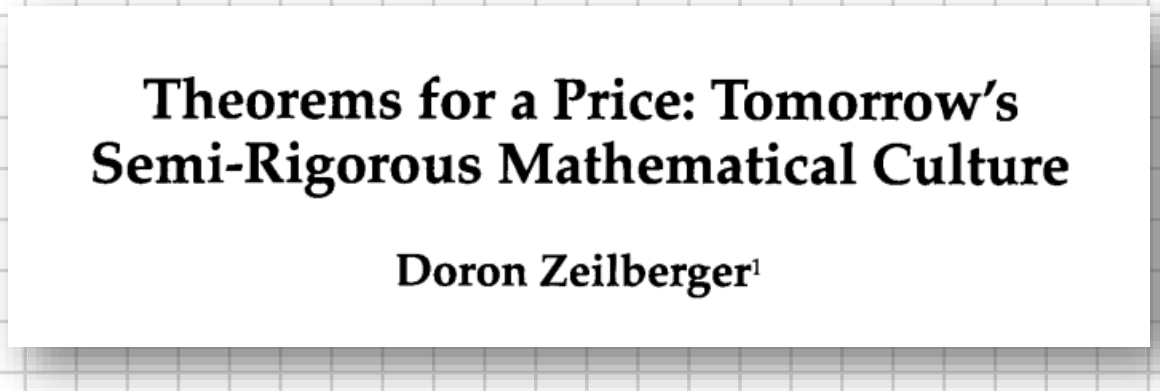


How we failed

Observations:

There is no ***5 sigma*** standard in math. If you want to publish a counterexample in a math journal you need a formal proof, not a probabilistic proof.

In our case, even if we found a modest counterexample on $n = 40$ vertices using Wagner's Algorithm, we would need at least $\$10^{20}$ to verify it via brute force.



Theorems for a Price: Tomorrow's Semi-Rigorous Mathematical Culture

Doron Zeilberger[1]

Submission history

From: Doron Zeilberger [view email]

[v1] Sun, 3 Jan 1993 20:34:57 UTC (9 KB)

Time in the wilderness

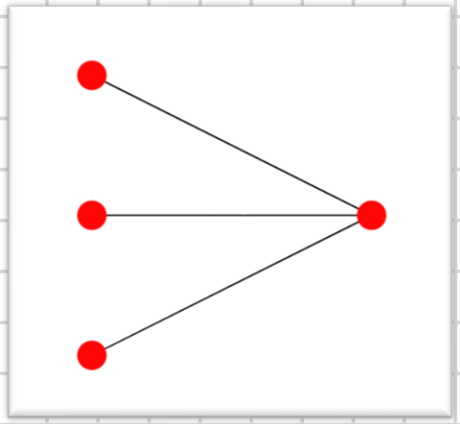
Observation: Bond percolation can be simulated by site percolation.

Theorem [Gladkov–Zimin'24]

Bond percolation does not simulate site percolation.

Theorem [Gladkov–Zimin'24]

Bond percolation does not simulate 3-hypergraph bond percolation.



Goal: *Resolve BBC for three pole vertices.*

Time in the wilderness

Definition [Wierman–Ziff hypergraph percolation]

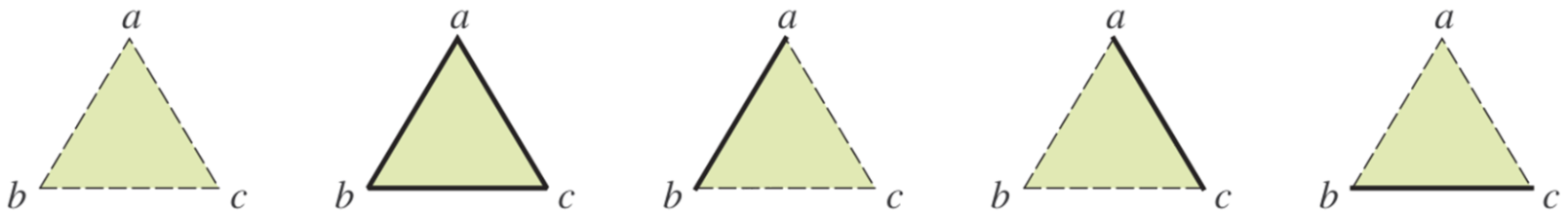
Condensed Matter > Statistical Mechanics

arXiv:0903.3135 (cond-mat)

[Submitted on 18 Mar 2009 (v1), last revised 12 May 2010 (this version, v2)]

Self-dual planar hypergraphs and exact bond percolation thresholds

J. C. Wierman, R. M. Ziff



$$p_{a|b|c} + p_{abc} + p_{ab|c} + p_{ac|b} + p_{a|bc} = 1$$

Question: *For which parameters the WZ hypergraph percolation can be simulated?*

[Gladkov–Zimin'24]: *Both necessary and sufficient conditions.*

BBC resolved

Theorem [Hollom'25]

The BBC is false for 3-hypergraphs.

Mathematics > Combinatorics

arXiv:2406.01790 (math)

[Submitted on 3 Jun 2024]

The bunkbed conjecture is not robust to generalisation

Lawrence Hollom

Theorem [Gladkov–Zimin'24]

Bond percolation does not simulate 3-hypergraph bond percolation.

BBC resolved

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Mathematics > Combinatorics

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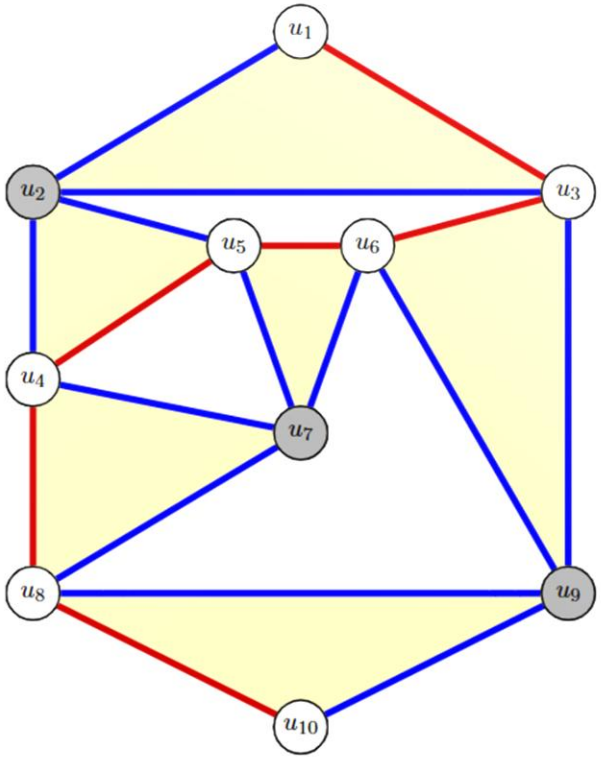
Lawrence Hollom

$$p = \frac{1}{2}$$

$$S = \{u_2, u_7, u_9\}$$



Main Lemma [Gladkov–P.–Zimin]

The BBC is false for the WZ percolation on Hollom's 3-hypergraph,

for $400\, p_{a|bc} \leq p_{abc}\, p_{a|b|c} - p_{ab|c}\, p_{ac|b}$.

BBC resolved

Main Lemma [Gladkov–P.–Zimin]

The BBC is false for the WZ percolation on Hollom's 3-hypergraph,

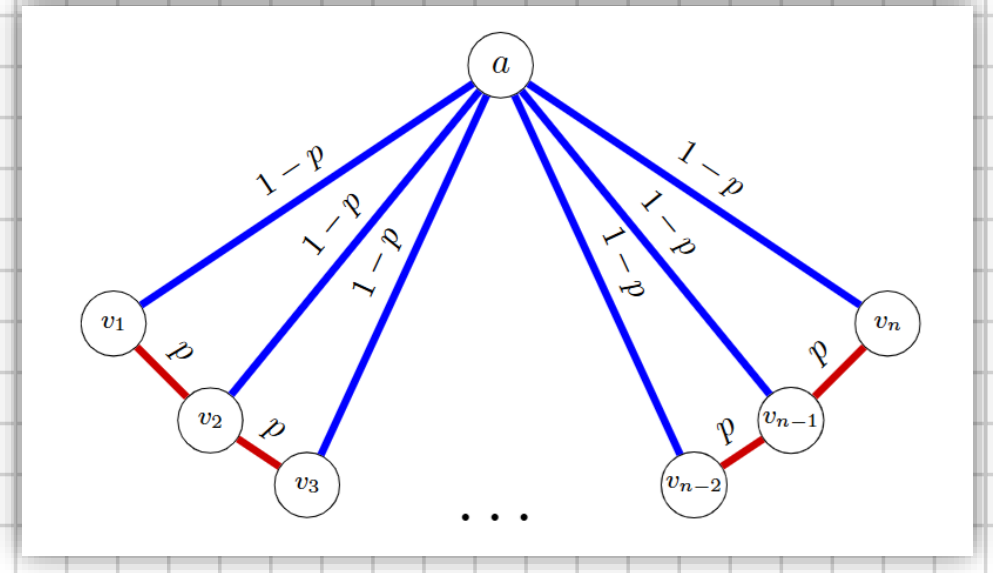
for $400\,p_{a|bc} \le p_{abc}\,p_{a|b|c} - p_{ab|c}\,p_{ac|b}$.

Proposition [Gladkov–P.–Zimin]

The following gadget simulates desired WZ probabilities.

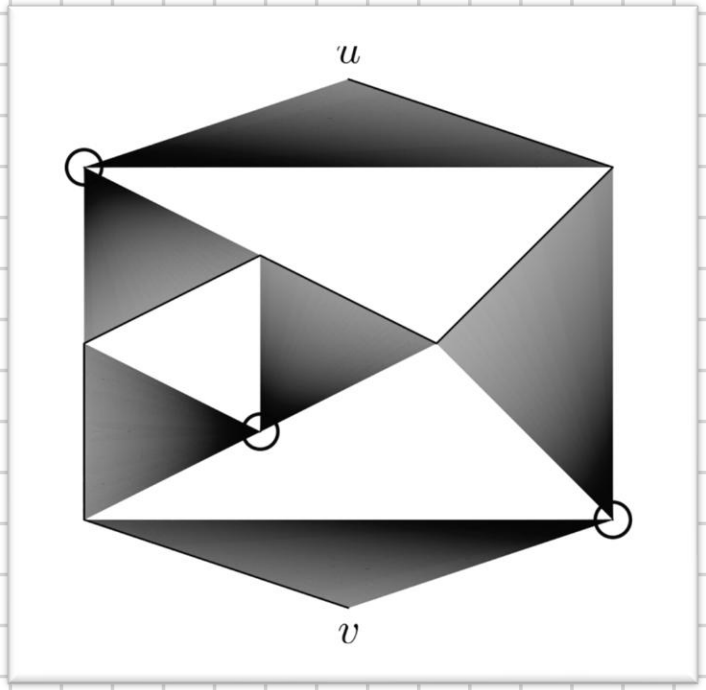
$$p = \frac{1}{2}$$

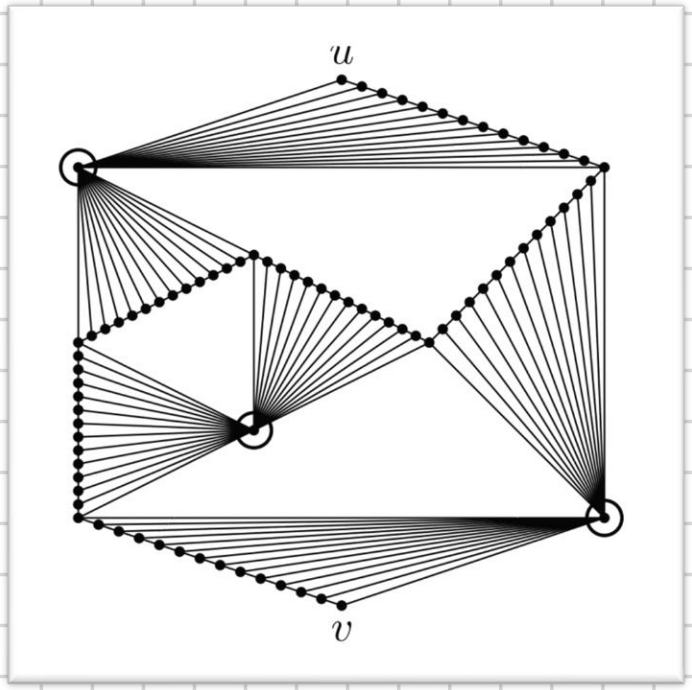
$$n = 1204$$



BBC resolved

Corollary: Replacing every hyperedge in Hollom's 3-hypergraph with the gadget gives a planar graph on 7222 vertices, which is a counterexample to BBC for $p = \frac{1}{2}$.





BBC resolved

Corollary: Replacing every hyperedge in Hollom's 3-hypergraph with the gadget gives a planar graph on 7222 vertices, which is a counterexample to BBC for $p = \frac{1}{2}$.

Remark: This counterexample has BBC probability gap $\delta \approx 10^{-4331}$. A computer assisted computation shows that one can take a smaller graph with only 82 vertices. This smaller counterexample has $\delta \approx 10^{-47}$, still out of reach computationally.

Lessons learned:

1) When choosing a problem, listen to the elders.

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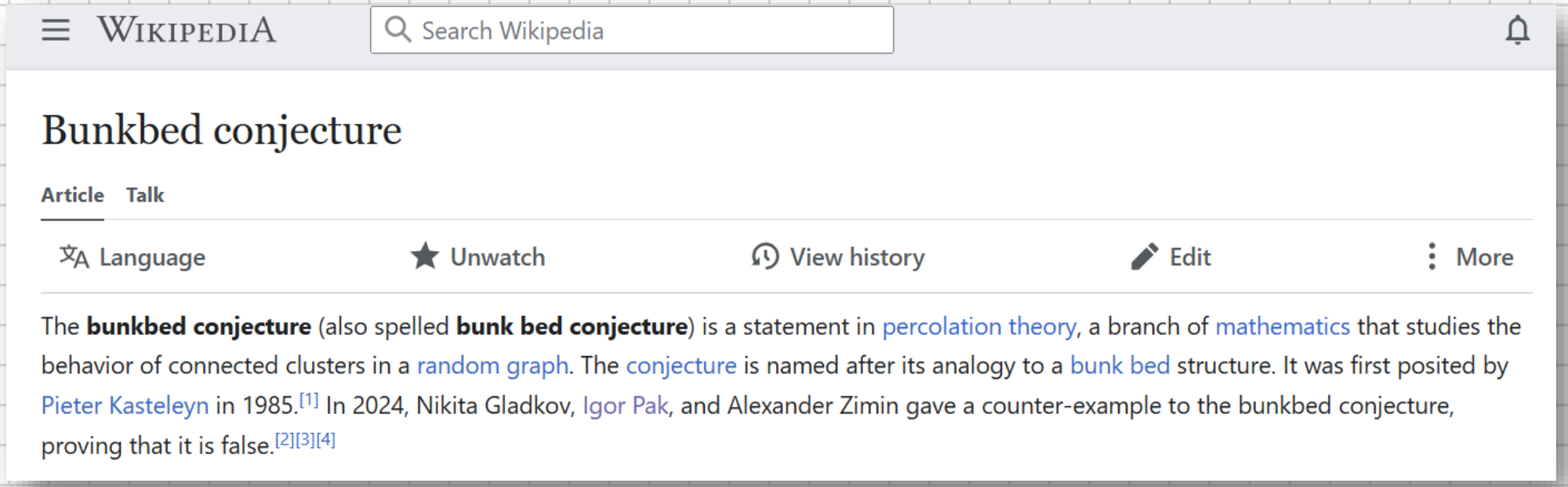

Lessons learned:

1) When choosing a problem, listen to the elders.
2) If you like a problem, create its *Wikipedia* page.
3) Read both stat physics and combinatorics literature.

Condensed Matter > Statistical Mechanics

arXiv:0903.3135 (cond-mat)

[Submitted on 18 Mar 2009 (v1), last revised 12 May 2010 (this version, v2)]

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Mathematics > Combinatorics

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Lawrence Hollom

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3) Read both stat physics and combinatorics literature.
4) Make a lot of experiments. Be thoughtful about them. Fail spectacularly. Write about them anyway.

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1) When choosing a problem, listen to the elders.
2) If you like a problem, create its *Wikipedia* page.
3) Read both stat physics and combinatorics literature.
4) Make a lot of experiments. Be thoughtful about them. Fail spectacularly. Write about them anyway.
5) When writing a paper, don't try to optimize your counterexample — the bigger the better!

7222 vertices!

Lessons learned:

1) When choosing a problem, listen to the elders.
2) If you like a problem, create its *Wikipedia* page.
3) Read both stat physics and combinatorics literature.
4) Make a lot of experiments. Be thoughtful about them. Fail spectacularly. Write about them anyway.
5) When writing a paper, don't try to optimize your counterexample — the bigger the better!
6) Be lucky!

Thank you!