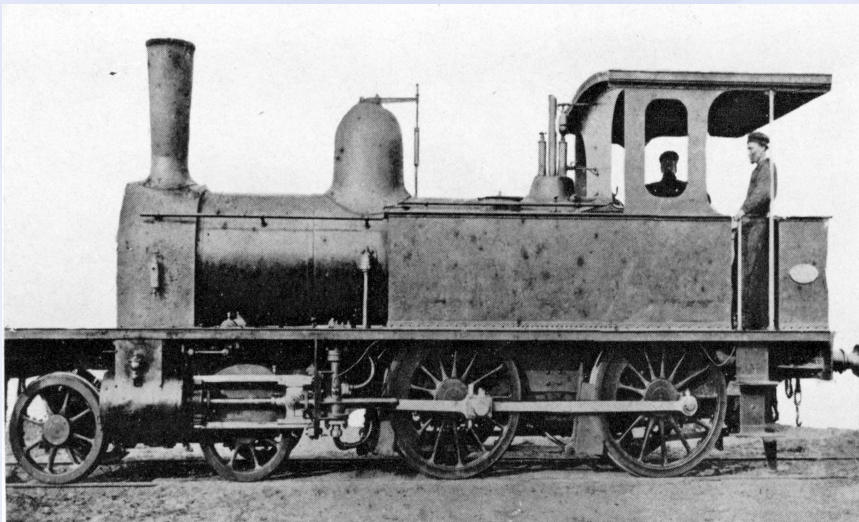


Geometry of mechanisms: from trains to computers

Igor Pak, UCLA



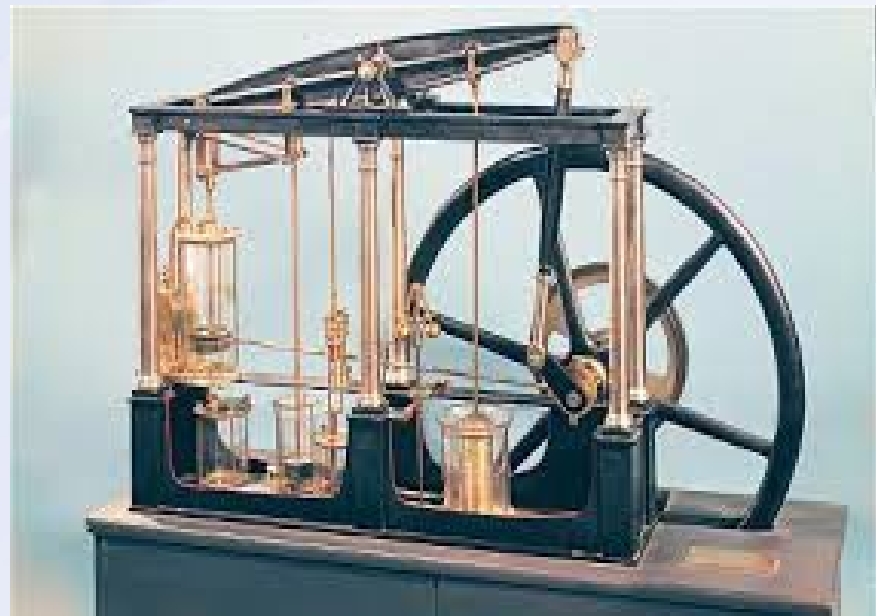
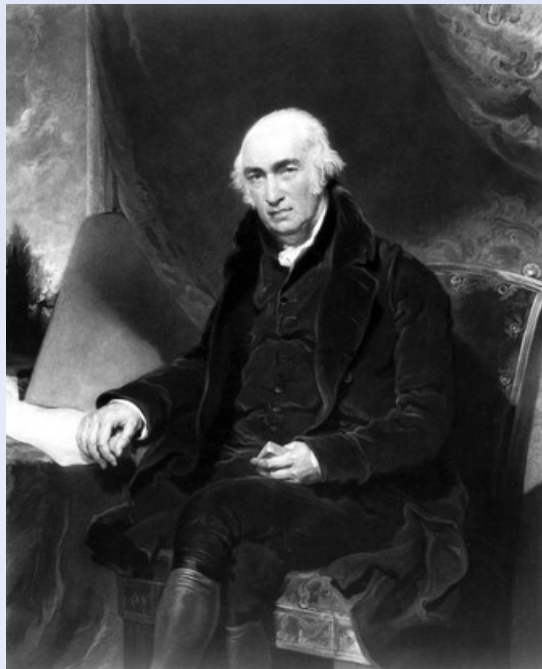
How linkages changed the world

- Watt's linkage
- Straight line linkage
- Kempe's universality theorem
- Planimeter
- Differential analyzer
- Shannon's digital circuits

But the real story is about inventors and their ideas!

James Watt (1736 - 1819)

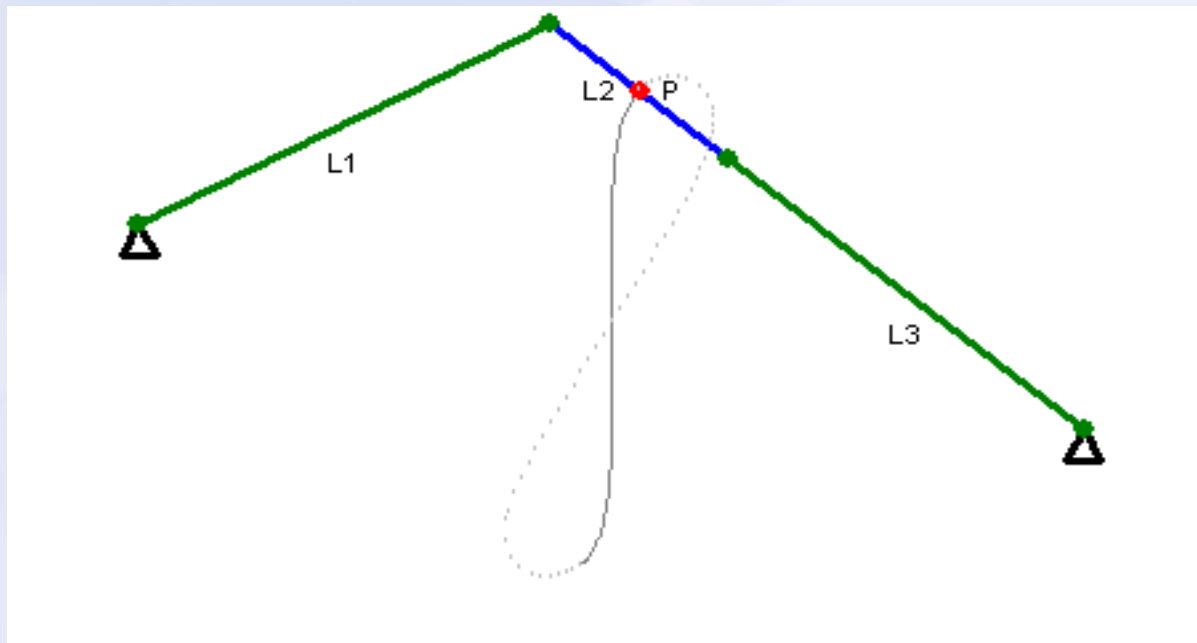
Scottish mechanical engineer, inventor of Watt's steam engine (c. 1775)



Watt noted: *Trains use wheels!*

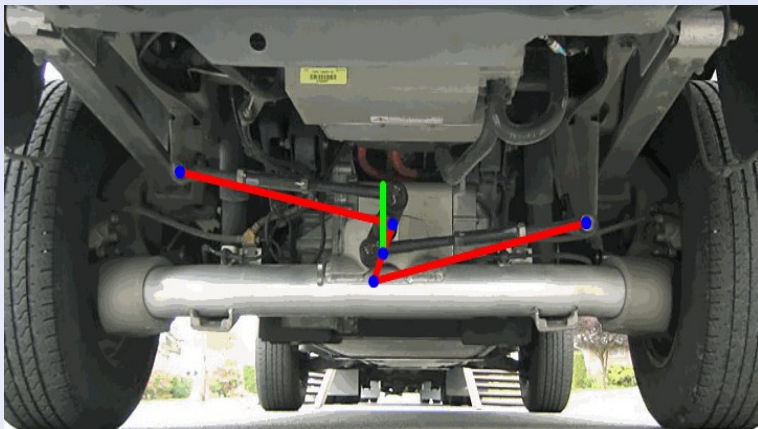
Problem: How to transform straight-line motion into rotary motion?

Eureka! Watt found a solution in 1785



Used in suspensions of:

- Ford Crown Victoria, Saab 900, Opel Astra
- Aston Martin, Alfa Romeo, Mazda RX-8



1998 Ford Ranger

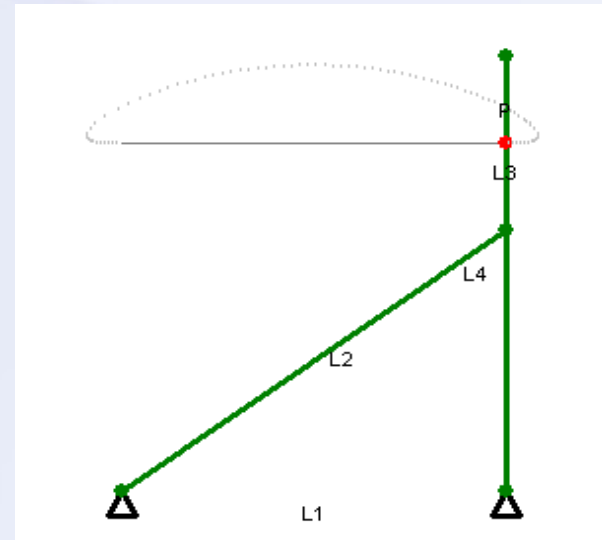


1964 Aston Martin DB5
in *Goldfinger*

Century long quest for perfect straight line linkage

Pafnuty Chebyshev (1821 – 1894), Russian mathematician

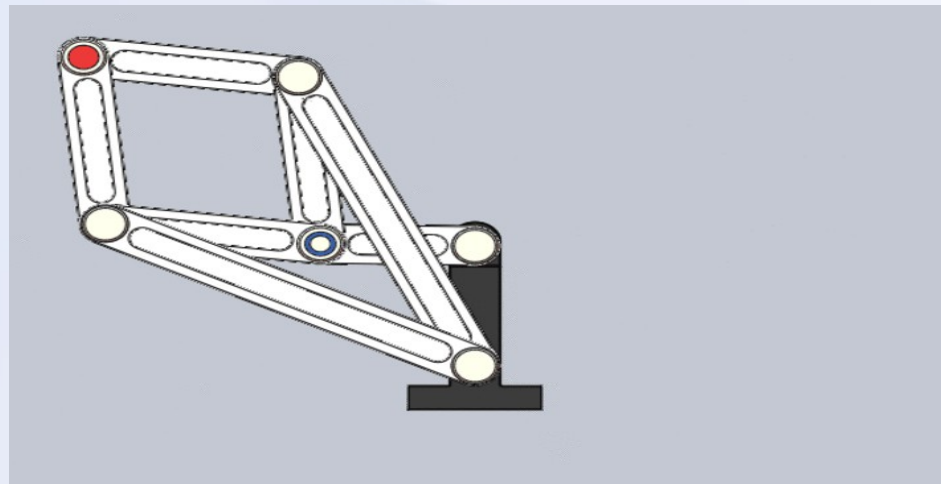
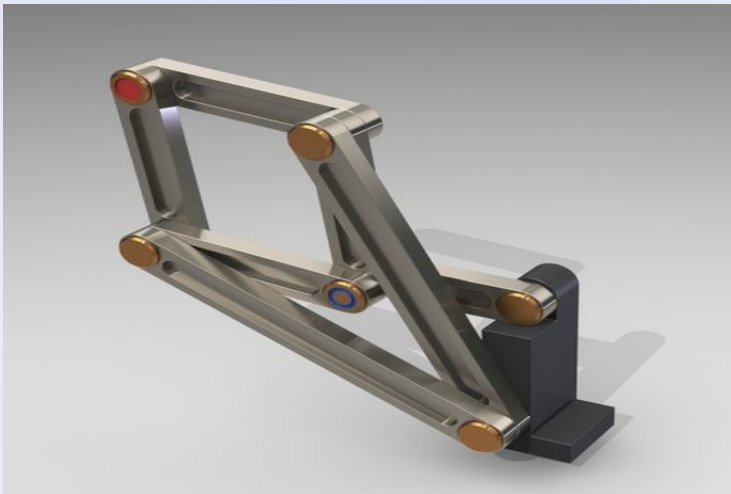
- was obsessed, believed this is impossible!
- invented Chebyshev polynomials to approximate a line



The Discovery:

Perfect straight line linkage was discovered independently (1864,1873) by two inventors:

- Charles-Nicolas Peaucellier (1832-1913, French army officer)
- Yom Tov Lipkin (1846-1876, Lithuanian Jewish mathematician)



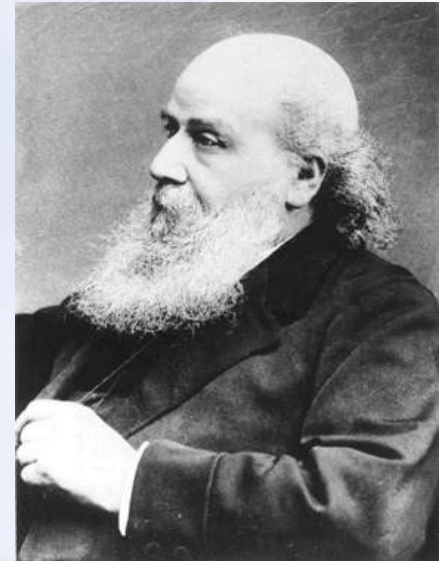
Famous seminar:



Arthur Cayley
(1821 – 1895)

[matrix guy, Cambridge]

May 1876



James J. Sylvester
(1814 – 1897)

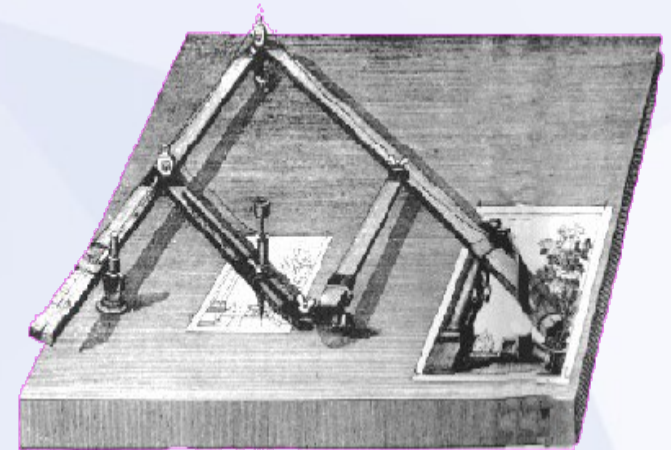
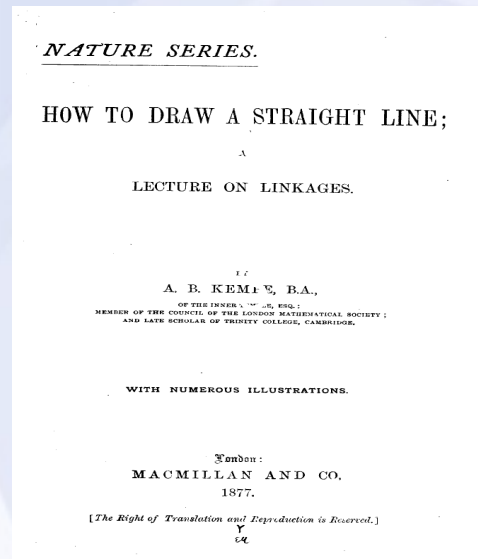
[1st American mathematician,
Johns Hopkins, Oxford]



Kempe's Universality Theorem

Alfred Kempe (1849 – 1922), London barrister

Kempe realized in 1877 that the straight line linkage + few more simple linkages allow drawing of *any algebraic curve*, i.e. solve **any system of algebraic equations**.



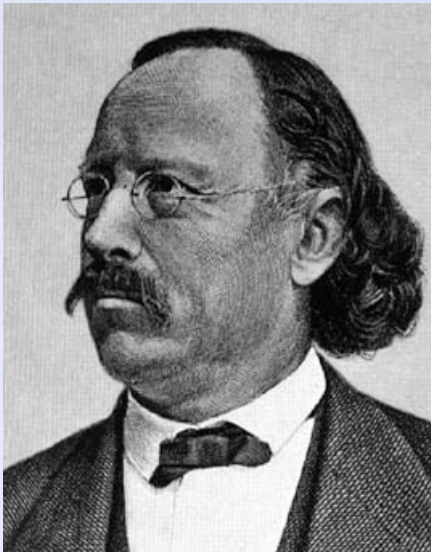
pantograph

Planimeter

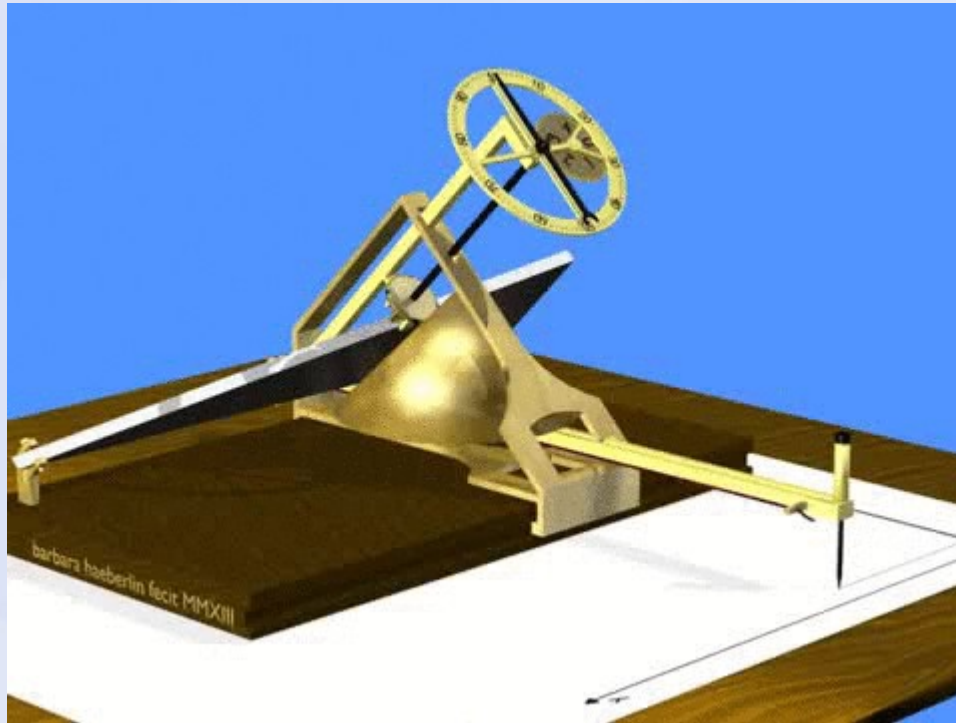
Discovered by a Swiss **Jakob Amsler-Laffon** in 1854.

Allows computing areas of curves in the plane.

Extended Kempe's theorem: combined, linkages and planimeters allow solving of **any system of differential equations**.



Another Planimeter



Differential analyzer

Vannevar Bush (1890 – 1974), engineer at MIT

Invented **analogue computer** which can solve high order ODE on demand (extending ideas of **Alfred Kempe**)

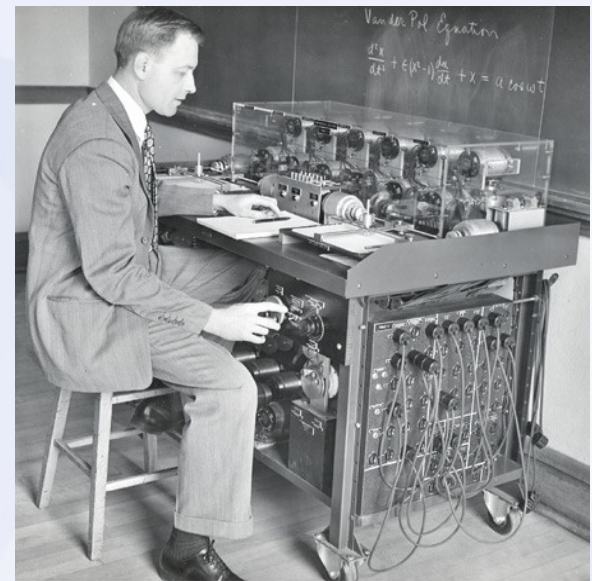


20 years of history of differential analyzers

- First practical model 1928-31 (6 mechanical integrators)
- Became obsolete in 1950 after invention of DDA
- The biggest and most expensive: GE built a DA for UCLA in 1947 (price: \$125,000)

Movie!

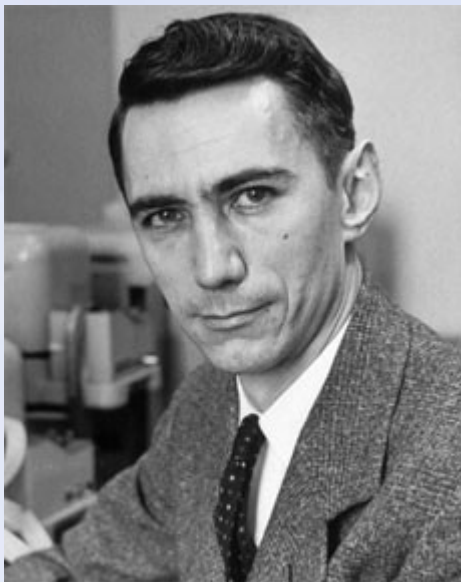
DDA



Digital computer

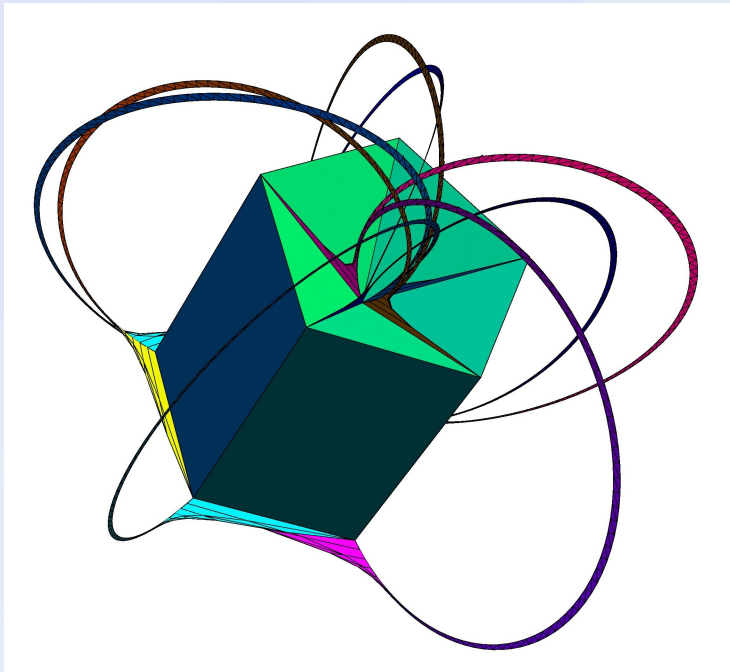
Claude Shannon (1916 – 2001), American mathematician

- Assisted Vannevar Bush at MIT to set up the machines
- Realized connection to Boolean logic (MS thesis, 1937)
- This led to the digital circuit design



Why I care

Together with Stedman Wilson, we made this
irrational arrangement of triangulat prisms:



1277 prisms

Solves $z^2 = 5$

Realizable over $\mathbb{R}$ but not
over $\mathbb{Q}$ (idea: [Kempe's thm](#))



Picture credits: Wikipedia, Google Images