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**Chan, Swee Hong** (1-UCLA); **Pak, Igor** (1-UCLA);
**Panova, Greta [Panova, Greta Cvetanova]** (1-SCA)
**Log-concavity in planar random walks. (English summary)**
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The authors study a certain exit probability of a nearest neighbor lattice walk in $\mathbb{Z}^2$ from a prescribed simply connected planar region.

The following is an abridged formulation of the discretized version of their result. Let $\Gamma \subseteq \mathbb{Z}^2$ be a simply connected planar region whose boundary is determined by two disjoint line segments, namely $\alpha$ and $\beta$ (viewed as perpendicular to the $x$-axis), whose ends are connected through two disjoint $x$-monotone curves. Let $\{X_n\}_{n\in\mathbb{Z}_{\geq 0}}$ denote a nearest neighbor random walk over $\mathbb{Z}^2$ starting somewhere on $\alpha$. The authors prove that

$$\Pr[X_T = (m,k)]$$

is log-concave, where $T$ denotes the first time the walk reaches $\beta$.

The proof techniques used in this article are riveting. *Elad Aigner-Horev*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*