From: Kung, Joseph <Joseph.Kung@unt.edu>
Date: Mon, Oct 18, 2021 at 11:24 AM
Subject: Re: question on Rota's conjecture
To: pak@math.ucla.edu <pak@math.ucla.edu>

Dear Igor,

[...]

Although Rota stated his unimodality conjectures in basically two bare sentences, the background to this is ``dense''. I had many a discussion with him, starting from my youthful challenge: why is unimodality even interesting... I am happy to see something like this in the first sentence of your paper: if almost every sequence is unimodal (and unimodality conjectures are made everyday like cats asking for food), why bother. Rota's answer is that unimodality inequalities are central to convex sets... they give isoperimetric inequalities for example. So he is looking at a wide range of results (the Brunn-Minkowski inequalities for example: how suitably refined unimodalitiy inequalities can lead to what are the ``extremal'' matroids and what they converge to: this is vague, a toy example is that the regular n-gons are the extremal polygons for certain isoperimetric inequalities, and the circle is what they converge to.... Rota was, in a way, looking for the circle... the infinite matroid-like structure that would allow a ``continuous matroid theory''.

We also discussed Steiner's theorem, and wonder if one can define a Minkowski sum of matroids or perhaps, antimatroids.... and so on. We never managed to do anything concrete with our discussions.

I cannot really provide you with a reference: these are conversations (over dinner or long walks in Cambridge and elsewhere). The closest reference is the short book he has with Dan Klain (Introduction to Geometric Probability), but one would have to read between the lines. In the book, they say that ``convexity is continuous combinatorics'', and you can cover a multitude of conjectures under that doctrine.

I had hoped that with retirement, I will write all this up in two papers: one on the finite minor conjecture and the other on the unimodality conjectures, but it is extremely hard... and I am not even sure that there is anything in this except as an amusing story about the fish that we never caught.

[...]

Best wishes. Joseph

From: Igor Pak <pak@math.ucla.edu>
Sent: Monday, October 18,2021 3:10 AM
To: Kung, Joseph <Joseph.Kung@unt.edu>
Subject: question on Rota's conjecture

Dear Joseph,

Hope you are doing well. I have a quick question. In §17.2 of our latest paper we mentioned your discussion of Rota's unimodality conjecture and how it's related to the mixed volumes. However, I could not find these Rota's ideas neither in his Nice ICM talk nor in his joint paper with Harper. Were you writing from

memory of private conversations with Gian-Carlo, or perhaps there is some other source I am missing? Any other comments or suggestions on the paper are also very welcome, of course.

Many thanks and best wishes, -- Igor

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Swee Hong Chan and Igor Pak,
Log-concave poset inequalities, preprint (2021), 71 pp.
https://www.math.ucla.edu/~pak/papers/research.htm#ot