CHAPTER X

On the Future of Combinatorics

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THIS PAPER was born with two strikes against it. First it is not a spontaneous outpouring of the heart. Rather, it was solicited from the author as part of a project in honor of Nick Metropolis birthday. Second, solicited papers on general topics usually manage to tell you nothing that you did not already know and yet, somehow, tell you more than you had hoped to read. Reading one is something like drinking a very large glass of club soda, differing mainly in that it can be recommended at bedtime to induce healthy sleep. Honoring Nick Metropolis is a duty I am compelled to perform. You may rejoice that you are under no similar obligation to continue reading.

My appointed subject is the future of combinatorics; I am informed that my comments will be published unexpurgated. This of course is a direct invitation for me to write something worth expurgating, and I will attempt to oblige.

To communicate anything reasonable on this subject requires raising and answering quite a number of related questions.

First, what is combinatorics?

I have never been able to find a satisfactory answer to this question. The term "combinatorics" seems to refer to discrete or finite aspects of mathematics rather than those involving continuity. This definition wont do. There are combinatorial aspects to all mathematical subjects especially analysis, and continuous methods are perfectly acceptable in combinatorics. Furthermore, there are fields that can be and are called combinatorial topology, combinatorial geometry, and algebraic combinatorics; logic and probability are highly combinatorial subjects.

In practice, my lack of awareness of any meaningful definition of the subject never bothered me. I have worked and continue to work on the problems I like to work on, and I call the results combinatorics. For me, the issue has only arisen at cocktail parties, when someone attempting to patronize me has asked what I do, or when I have tried to compose documents like this one.

This is the case despite the fact that there is a perfectly good definition of the subject, or rather of the concept of a *combinatorial argument*. Combinatorics is the area of mathematics that is concerned with, relates to, employs, studies combinatorial arguments. So our second question arises: what is a combinatorial argument?

Explored, known mathematical results can be pictured as a huge imaginary edifice of truths all derivable from one another and from certain original principles. Mathematicians typically try to extend this structure, or apply parts of it to the real world, or both. Such efforts consist of arguments, every argument containing two components: knowledge about the existing structure of mathematics, and *ingenuity,* including complex, devious, and perhaps very detailed and complicated reasoning.

A *combinatorial argument* is one that consists predominantly of ingenuity or detailed reasoning rather than knowledge of existing mathematics. This is in contrast to a *knowledge-based* argument, which relies heavily on piecing together known results. An argument of the following form is not combinatorial: by the theorem of X we have A; by the theorem of Y we have B; from A and B we get the desired conclusion. This kind of argument is based heavily on knowledge of the theorems of X and of Y. It could, however, be a combinatorial argument to you if you were ignorant of the theorems and recreated them yourself.

It is obvious why combinatorial arguments and, by extension, combinatorial subjects were never popular among traditional mathematicians. They spent much of their lives learning lore about their subject. Combinatorial arguments tend to ignore this knowledge. Combinatorics can be attempted by inspired amateurs and dilettantes who know little of formal mathematics, and much of it was developed by such people. One goal of formal mathematics is to systematize the structure of mathematical knowledge to abolish the need for intricate reasoning and thought. Combinatorics therefore represents the areas of mathematics that are failures in this sense, those for which systematic developments of past knowledge are relatively useless in attaching present problems. At one time combinatorics was derisively referred to as the "slums" of mathematics.

Of course, talented mathematicians in all fields routinely use and invent combinatorial arguments. They are honorary combinatoricists, differing from true combinatoricists mainly in that the problems they work on are far more heavily imbedded in the structure of mathematics than are most combinatorial problems. You need lots of knowledge in order even to understand what they are.

To understand the role of combinatorics in mathematics, it is helpful to keep two analogies in mind. First, combinatorics and combinatoricists play exactly the same role in mathematics and among mathematicians as mathematics and mathematicians play in the sciences and among scientists. Second the position of combinatoricists in mathematics, and mathematicians science, in many ways parallels the position of Jews in European society in days gone by.

European society tended to be hierarchical and organized. Trades and occupations were controlled by guilds that excluded outsiders. Jews were disliked and even hated for reasons that include the following:

1. They were different, an alien element in society;

2. They had a reputation for cleverness;

3. They were without much power or clout, so they could be sneered at or assaulted without risk or cost.

4. Excluded as they were from all established occupations, they were always heavily represented in new pursuits that changed society; they came to symbolize everything that had destroyed the idyllic "good old days" and threatened to make modern life unstable. Once a new pursuit became established, they were excluded from it.

5. Some individuals felt a special need to avoid identification with Jews, and so took a special interest in attacking them. For example, Richard Wagner was born in a ghetto, looked Jewish, had many Jewish admirers and had a Jewish-sounding name before he changed it to Wagner; Karl Marx and Ludwig Wittgenstein were of Jewish descent; Luther and Mohammed created their own religions that had to be distinguished from Judaism. All of these men were outspoken haters of Jews.

6. There was a religious, or, rather, anti-religious, component, as well: Jews were resented for and held responsible for the Ten Commandments, particularly by those who had difficulty obeying them.

The first three elements on the list clearly characterize mathematicians among scientists as well as Jews in European Society. That there is an exclusion of mathematics from science can be illustrated in the following way:

Mathematics is of great importance, of use in all sciences. Quantitative physics, for example, required calculus for its formulation, and its development, including the industrial revolution, which it spawned, could not have taken place without the development of calculus. Much of mathematics was and is motivated by potential application to science or engineering.

When such an application is successful, it quickly becomes absorbed into the subject; if important, it becomes a part of the science, is taught to students as such, and ceases to be considered mathematics. Thus, what is left as mathematics, and taught as such in areas of application, are either preliminaries or those subjects that have not been successful enough or important to be taken over. Applied mathematics is, in effect, excluded from its own successes.

Of course, there is always the possibility that new mathematical methods will be required to handle new or not yet understood scientific problems. Thus mathematicians represent the threat to scientists that they may be forced to learn new tricks and to study new and perhaps unfathomable mathematical lore.

There are analogs of even the last two entries on our list. Individuals who specialize in the theoretical aspects of science sometimes find it necessary to distinguish their own works from mathematics by disparaging the latter. Finally, there is a mathematical analog of the Ten Commandments resentment. This is an adult version of the childhood hatred of the imposed discipline to virtue. Arithmetic is another hated discipline imposed upon and resented by children, and mathematicians are similarly blamed for it.

That combinatorics produces resentment of the first three kinds is not at all surprising. What is perhaps most interesting in this analogy is that combinatorics also eventually becomes estranged from and separated from its successes. That this happens stems from a self-destructive feature of its existence which is extremely important for understanding its future.

When research is performed successfully in any area, results are obtained and knowledge is developed. Eventually all the relatively easy and clearly important results have been discovered, and what is left is difficult or obscure or special or, on first glance, uninteresting. The more we discover, the less is left to find. This development of knowledge is inevitable in all fields, and tends, eventually, to make subjects that have been long and heavily studied forbidding. As a subject matures in this way it becomes necessary for a newcomer in that field to learn more and more existing results to develop the ability to attack problems of less and less interest. In a combinatorial subject, the situation is worse: as it develops, knowledge of past results inevitably grows in importance, eventually becoming as important as ingenuity, and the subject ceases to be combinatorics at all, receding into just one more area of mathematics.

This is how research in combinatorics leads inevitably to its own destruction. Its fate, like yours or mine, is certain death.

Fortunately for us, this is an answer only for the long run. We may safely ignore it in our lifetimes. The future of combinatorics in the short run comprises three distinct questions: first there is its future as a research subject; second, its future as a component of mathematical education; and, finally, its future as an art form. To understand these matters, one must have a picture of the past and present of combinatorics in these contexts. To describe these I will review the past and present of mathematics and, indeed, of physical science itself.

Modern physical science dates from the development of calculus by Newton and Leibnitz in the 1600s. It was a European phenomenon, centered in England, France and Germany, and wherever similar universities existed in Italy, Austria, Hungary, Russia, Sweden, Switzerland, Poland, etc.

The United States was a provincial backwater in the field of physical science. There were occasional important figures, such as Franklin, Sylvester, Gibbs, Michelson, Veblen and Wiener; in exactly the same sense, Denmark had the even more important figures of Brahe, Oersted and Bohr. This was in contrast to technology; since the mid-nineteenth century the United States has played a significant role in technological development. Even here, initial inventions were often European, but industries and mass markets, and the technological improvements that came with these developed most rapidly in the United States.

This situation continued until the 1930s, when we received an influx of scientifically eminent refugees in particular Jews and socialists fleeing Hitlerism. Many of these eventually found positions at American universities. This immigration (including, for example, Bethe, Einstein, Fermi, von Neuman, Teller, Wigner, and Ulam) did not in itself raise the United States to its present position in the field of physical science and mathematics, but it did make the country into a world-class player in science, and was helpful in providing as infrastructure for the developments to come.

To understand what happened next, you must be aware of the scientific and technical situation in World War II, and I will digress to describe it. When the United States entered that war, the weapons systems and military tactics were, in many key respects, significant, inferior to those of the enemies. This inferiority contributed to heavy casualties among U.S. troops, and greatly encouraged the enemy and in particular, the Japanese attack and Germanys declaration of war on us.

1. U.S. aircraft were significantly inferior to German and Japanese aircraft at the beginning of the war. (The first planes were no match for the Japanese Zeroes and were shot down in droves. This fact was withheld from the American public through censorship for morale reasons, and was never subsequently publicized.) The U.S. was able to catch up with the Japanese, and eventually to dominate the skies, although the Germans retained a technological lead (developing combat jet planes and rockets), which, surprisingly, they were unable to exploit effectively. As a further example, one of the most effective fighter planes, the P-47, was (according to British claims) entirely a British design simply manufactured in the United States.

2. Early in the war, U.S. torpedoes were largely ineffectual due to design defects. Defense against submarines was close to non-existent at the beginning. The Germans developed the "snorkel" before the U.S.

3. American tanks were small and under-armed, and thus were vulnerable to and impotent against the larger (and almost equally maneuverable) German tanks, unless attacking them from behind.

4. U.S. Infantry armaments and tactical doctrine were based on the remarkable success of American arms in World War I but were no match for the German arms and tactics. (These were developed between the wars in cooperation with the Russians.) Specifically, American infantry arms, mainly rifles, were outstanding in dealing with targets that were within sight. The German infantry used equipment, principally mortars, that was effective at longer distances. Thus in an equal combat, German infantry could inflict heavy casualties on U.S. forces at distances at which very little harm could be done to them. Something like 80% of U.S. infantry

casualties came from mortar fire. The potentially devastating effect of such disparity could be seen in the more recent Iraq in Kuwait campaign, when the advantage was on the side of the United States and in the success of the German "blitzkrieg" in Poland and France in the early stages of World War II.

5. American tactical air support of ground forces, while occasionally very effectively, was poorly coordinated, and often destroyed forward U.S. forces as well as forces of the enemy.

Under these circumstances, it is almost amazing that the two major battles that the United States fought with the Germans (Normandy and the Bulge) ended as lopsided American victories. In Normandy, naval gunnery could trump the mortar card; in the Ardennes, the utter failure of the German attack on the artillery on Elsenborn Ridge (itself produced in large part by a new secret weapon) condemned their entire operation to fatuity.

To be sure, the United States had superiorities over their enemies, particularly in radar, sonar and atomic weapons. All of these, along with enormous war production and supply capabilities, and the total command of air and sea that the U.S. achieved were important components of victory. The deficiencies as just described greatly increased the human cost of this victory.

This history, and the devastating and decisive impact of nuclear weapons, made it clear to U.S. military leaders after WWII that they could never again afford to fall behind potential enemies in military technology or tactical matters. To achieve this, the military made a conscious decision not only to spare no expense to stay at the forefront of technology in weaponry, but to support and maintain an infrastructure in mathematics and science in the United States that would be second to none in the world. The U.S. had to develop the best possible military technology. Having the lead in basic research was judged to be essential for them to do so.

The dramatic effectiveness and mystery of the atomic bomb also stimulated the imagination of a generation of young Americans who began to take an unprecedented interest in science and mathematics.

The United States has for a long time been home to a remarkable number and diversity of institutes of higher learning. These provide a market for academic scientists and mathematicians far greater than that in other countries, where education is mainly government run. When the importance of science became evident, a large number of positions at such institutions became available. Fortunately this happened when the scale of higher education was rapidly expanding.

The net effect of the difficult conditions in Europe and the restricted opportunities for young scientists there was little short of amazing. In a few years, the United States went from being a backwater to being the center and focus of research in the entire world. By (roughly) 1960 the United States had come from a bare presence to leadership and to the point where more than half the research in mathematics and physical science was performed in this country. For example, I am told that Nobel Prize winning by individuals born in the United States went from practically none to more than half in almost no time, and have stayed at that level ever since.

Since the 1960s, research in mathematics and physics has flourished in the United States on an unprecedented scale and basic research has been predominantly conducted in American universities. In the past it was advisable and perhaps necessary to know French and German to keep up with current research; now almost everything is published in English, and knowledge of other languages is of only slight utility. When I was a student, it was considered wise to spend a year or two in Europe for seasoning after receiving ones Ph.D. It is still sometimes done, but it is now considered frivolous. The places to go as a Post-Doc are mainly in the United States. Now, by contrast, the German government maintains a research laboratory in computer science for the benefit of German Post-Docs in Berkeley, California, and there is a laboratory in the same field in New Jersey sponsored by Japanese interests.

Of course, mathematics has prospered in the rest of the world as well. The mathematics community in western Europe has grown in tandem with the growth of higher education in general. Eastern Europe and the Soviet Union greatly encouraged the development of science and mathematics just as the United States did, but they follow a different course. The most coveted positions were in research institutes rather than universities. This put their fates entirely at the whims of bureaucrats. This was fine until the 1960s when these institutes reached saturation. The Soviet solution in mathematics was to employ the standard European version of "affirmative action," more properly called "negative action": exclude Jews who had represented almost half of Soviet mathematicians from entry into mathematics. (In the United States there is now an analogous movement in Congress to fight potential academic job shortages by cutting down on employment of aliens.) The government also crippled research by keeping a strangle hold on communications. Nevertheless, there was and is an impressive level of mathematics in the former Soviet Union and western Europe in general. The institute organization, however, made the system absolutely dependent on government subsidy. As various governments lost the will to fund mathematicians, these subsidies have dwindled to starvation levels. Mathematicians who can make arrangements to spend time in the United States or elsewhere in the west, and many have taken jobs in the United States.

In short, for the thirty or so years since 1960, the field of mathematics has enjoyed a prosperity and level of activity in America hitherto unknown. Though the government supplied an initial and important impetus to the growth of the mathematics enterprise, its continued health has depended more and more on the growth in need for mathematics in the real world, fueled by the computer revolution. Computers now allow mathematics to be used to solve problems in every field of human endeavor, on a time scale that makes the solution of practical importance. This is and will be incorporated into countless devices, whose development will require input from individuals with the skills of PhDs in mathematical science or physics.

The computer revolution itself was set into motion and for a long time sustained by the needs of our military efforts, which provided vital markets for computers when they were expensive and paltry by present standards. Now that computers are available, powerful and cheap, the need for government support of their development has vanished. (In fact, our tax laws now discourage computer purchases by individuals through their depreciation rules.)

The key question about the future of mathematical research is whether it can be weaned from the government support teat and survive on its own, the way computer has.

The prosperity of the United States mathematical enterprise since the 1960s has not been without its vicissitudes, foibles and weaknesses. I will review some of these.

The prevailing attitude in the United States much as we may regret it, has always been in accord with the famous dictum: "Those who can, do; those who cant, teach." Yet the attitude in the academic mathematics community, quickly communicated to the student body, is precisely the reverse: "those who can, teach; those who cant do." This attitude may have been helped by the way research support was funneled through academic organizations, and may have been helpful in boosting academic morale, but it is as unpleasantly arrogant as the other is insulting and demeaning. This attitude, and the availability of government research funding, has tended to estrange the mathematics community from the world of do-ers and from other potential sources of financial support. Thus, while mathematicians tend to respond heroically when asked to help on real world problems they have almost no skills at acquainting potential users of their availability for help, or at dealing with the real world.

The job market for academic mathematicians has weathered a wide variety of blows but has, so far, survived them all. First came the transition from expansion to a steady state, then the disappearance of the 65 retirement age, the sudden deluge of Chinese PhDs when President Bush allowed them to stay, the recent and current migration from Eastern Europe and, most recently the disappearance of retirement at age 70.

The United States government has a strange tendency to react to a perceived problem by taking action in a way that precipitates the very disaster it was presumably trying to prevent. (Thus in 1929 the Federal Reserve Bank felt that the stock market was too speculative and too high. In response it raised interest rates. The effects of this policy reinforced the natural corrective action of the market, creating the crash that led to the subsequent Great Depression. Unfortunately, its cure for prosperity was far more effective than its cure for depression.)

In the late 1960s, the NSF perceived that the academic market for faculty was facing transition from expansion to a steady state. It saw that the number of academic job openings would be quite small until the first generation of researchers started retiring in the 1990s. In consequence, it cut its support for mathematics, cutting out many postdoctoral research positions from grants and contracts, hereby precipitating the job shortage it was supposed to ease. That job shortage disappeared rather quickly at that time, as the ever-growing market for computer experts was able to absorb mathematically skilled people at all levels of education. Thus, the problem it had sought to address turned out to be fictitious, and the only job shortage was the temporary one that government itself had created.

This government reaction is quite typical; now that there is a tight academic job market again, research contracts are sliced even thinner. Fortunately, jobs keep appearing in non-traditional areas of the economy that relieve the job situation naturally. Right now, financial institutions, faced with problems of evaluating and pricing derivatives and other financial instruments, are looking for talented mathematicians.

Of course, we are in the midst of the computer revolution, which has yet to have its full impact on the mathematics and educational communities. Our need for secretarial support has declined drastically as we have learned word-processing. We communicate with wonderful efficiency via e-mail. The worldwide web and similar developments threaten to replace libraries and even journals. Eventually our lectures and courses will change, as will our entire educational enterprise. Perhaps the need to travel to meetings to renew mathematical contacts will disappear, along with the isolation of being at a school with no other mathematicians in your field.

What, then, will the future bring?

The future for mathematicians in the real world seems bright. Despite the fact that successes always remove the need for mathematics, new problems keep arising to replace those that disappear, and will continue to do so as long as the power of mathematics is recognized.

Combinatorics will survive as long as mathematics does. Some of it, particularly algebraic combinatorics, will develop into an ordinary legitimate branch of mathematics (and has essentially done so already). Others will be (and have been) incorporated into applied fields like operations research or theoretical computer science. But the real world will still create variations of problems, which will require new methods and ideas, and these will qualify as combinatorics in all respects. It is probable that the United States will be able to retain its primacy in combinatorics, in mathematics and in dependent technologies only by continuing to absorb talent from all over the world. If it ceases to attract talent, or excludes it by law, it will slide back toward obscurity.

Academic mathematics seems destined to derive its resources almost exclusively from its educational mission; mathematics will therefore stand or fall with the great institutions in which it mainly resides. Federal support has become quirky, unreliable, laced with idiotic bureaucratic rules; it will soon be more trouble than it is worth. This is not because such support is not valuable to mathematics and to society; it is both. It is even salable to the American public, as an investment in education and infrastructure. But it is now only a tiny part of a large scientific package, in a world in which anti-science has captured our media and much of our elementary educational establishment. It is not clear how much longer we can recruit mathematicians and physicists from our bright young people, or that Congress will continue to support it. Anti-alien bills from Congress could help our higher educational system implode along with the dominant place we now hold in technological development. (Right now government bureaucrats have, for no apparent reason, taken aim at tuition payment on behalf of graduate students. What they will attack next is anybodys guess.)

Thus the future of mathematics (and therefore of combinatorics) in the United States depends heavily on the future of higher education and of higher education in mathematics in particular. Unfortunately, presenting a full picture of the past, present, and future of education in the United States would require much more space than I can hope to usurp here. I will observe that public high school education has gone into a terrible swoon (United States students are close to last in the world in accomplishment), and the liberal arts at university levels are dissolving into incoherence. The education community is attempting new approaches to education, which will probably end as disasters. (Taking into consideration the ability of calculators to perform the arithmetic and algebraic tasks that we have always drilled into the heads of our children, they wish to concentrate mathematical education on learning ideas, and learning to use these tools in exciting ways, rather than on old-fashioned drill. The resulting change will undoubtedly work for some and fail for others as is the fate of all reforms. Typically, success is achieved among the brightest students and among those with inspired teachers. Educational reforms always fail among students who pay no attention. Since we now officially encourage inattention by paying children on welfare life pensions if they can qualify as unteachable, failure among many of these is almost guaranteed.) Our sole surviving remnant of excellence lies in our undergraduate and graduate schools of science and mathematics, and our government is taking aim at these. Yet there is, as usual, some hope for the future, and hope for positive change lies mostly in combinatorics, for it is the kind of real mathematics most accessible and most exciting to the beginner.

There is one other direction in which combinatorics can develop in importance both within mathematics and without, and that is in expanding the notion of mathematics as a kind of aesthetics, as an art form. Research mathematics has been present on the American scene for only a short time, as we have seen. It has had some impact on science and technology and continues to do so, but has as yet had no effect at all on the softer side of intellectual life, that is, on the arts.

Music and visual arts are closely related to mathematical and geometric designs, as is evident from the structure of Bach fugues, change ringing of bells, the similarities between certain abstract art and the design patterns on VLSI chips, and relations between topological and other concepts and the distortions of perspective created by, for example, Hogarth or Escher. And there is interest today in computer music, and there are interesting challenges (can one teach a computer to write an aesthetically pleasing fugue in the style of Bach?). We are today developing new areas of animated art.

All of this may contribute to the arts and may help to shed light on the nature of the aesthetic experience in these areas, but all are still within the traditions of established arts as we know them. I want to emphasize something entirely different: an aesthetic based not on the senses of hearing and sight, as are music and art, but on the inner workings of the brain, on reason.

This is not a new concept. Literature is usually encountered through the eye, by reading, but its impact is interior, on the brain and its store of information about our experience. Our enjoyment of it is entirely intellectual and bears no direct connection to the senses. Still closer to mathematics, we have the aesthetics of the chess problem, or the go problem, or of the chess combination or of a coup in bridge. Each of these has an appeal to our sense of intellectual design and to our appreciation of efficient use of minimal resources to accomplish a difficult end. These are all miniature art forms, whose appeals are unfortunately buried in very specialized contexts, inaccessible to all but the experts, or, at the very least, the initiates in the corresponding games.

Mathematics, and, more specifically, combinatorics, is full of intellectually beautiful arguments and structures. These constructs are an important part of the appeal of mathematics to mathematicians. They represent an important aesthetic resource of mankind, yet they lie hidden to almost everyone outside mathematics, and to many within it. That mathematics is a realm of magnificent aesthetic joys is one of the best-kept secrets of the 20th century.

Most Americans have no clue that there is aesthetic enjoyment to be derived from mathematics, or that they could derive pleasure from exposure to mathematical ideas. In the 19th century, our newspapers and magazines sometimes presented mathematical problems, or ran chess columns, far more so than today. In recent memory, Martin Gardner ran a highly successful mathematical column in Scientific American for several decades, in which he tickled the taste for mathematical beauty. In Japan there are popular mass consumption mathematics magazines with circulations, I am told, in the hundred thousands. Yet the average American views mathematics only as something horrible which defeated him/her at some stage in his/her life.

Is there a future for mathematics as scope for aesthetic experience? Today we see little sign of it. Beautiful arguments abound, but they are presented only to technical audiences who usually care more for the results than for the arguments. Combinatorists, and mathematicians in general, like the Jews, are not proselytizers. They keep their mathematics to themselves, making no effort to popularize their arguments or develop an audience for them. They use notations and jargon that keep their ideas impenetrable to the uninitiated.

In the 1930s there was no sign that mathematics in the Untied States would ever emerge from obscurity, but then it did. Perhaps mathematics, and in particular combinatorics, needs only a Messiah and a Paul to spread its joys to the unbelieving nation. We pray for their coming.

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