SOLUTION 2 (18.05, SPRING 2003)

1.1-4 a) and b) – straightforward calculation

c) The mode is 9 with .32 frequency

1.1-6 a) and b) – straightforward calculation

1.2-2 Use calculation of frequencies in 1.1-6 to obtain $\mu = 8.18$.

1.2-6 a) $\mu = \frac{4}{3}$.

b) Var$= \sigma^2 = 1\frac{2}{9}$.

1.2-8 a) $3, 19, 16, 9$, the total is 47.

b) $\mu \approx 2.66$, Var$= \sigma^2 \approx 0.73$.

c) – straightforward.

2.4-2 a) $P(A \cap B) = 0.3 \cdot 0.6 = 0.18$ by definition of independence. Thus $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = 0.72$.

b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0.6} = 0$.

2.4-8 Denote this event by Z. Observe that $P(Z) = P(Z_1) + P(Z_2) + P(Z_3)$, where Z_1 is an event of A being *not orange* and dies B, C being orange, Z_2 is an event of B being *not orange* and dies A, C being orange, and finally Z_3 is an event of C being *not orange* and dies A, B being orange. We have:

$P(Z_1) = \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{5}{36}$

$P(Z_1) = \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} = \frac{2}{36}$

$P(Z_1) = \frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{1}{36}$

And finally $P(Z) = P(Z_1) + P(Z_2) + P(Z_3) = \frac{5+2+1}{36} = \frac{8}{36} = \frac{2}{9}$.

2.4-10 Denote by X_1, X_2, X_3 the outcomes on dies D_1, D_2, D_3. Then:

a) $P(A) = P(X_1 = 3)P(X_2 = 2) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$.

b) $P(B) = P(X_2 = 2)P(X_3 = 1) + P(X_2 = 5)P(X_3 \le 4) = \frac{3}{4} \cdot \frac{2}{4} + \frac{1}{4} \cdot \frac{3}{4} = \frac{9}{16}$.

c) $P(C) = P(X_1 = 0) + P(X_1 = 3)P(X_3 \ge 4) = \frac{1}{4} + \frac{3}{4} \cdot \frac{2}{4} = \frac{4+6}{16} = \frac{10}{16}$.

2.4-12 a), b), c), $P = \frac{1}{2^5} = \frac{1}{32}$.

d) $P = \binom{5}{3} \cdot \frac{1}{32} = \frac{5!}{2! \cdot 3!} \frac{1}{32} = \frac{120}{2 \cdot 6} \frac{1}{32} = \frac{10}{32} = \frac{5}{16}$. (this is the probability of *exactly* three heads in five trials).

2.4-16 b), P(you win)=P(WIN comes first, third, fifth) $= \frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$.

a) **(credit for this part of the problem will be given to all–we didn't have time to cover this mater before Friday).** For your help, here is the solution:

P(you win)=P(WIN comes first, third, fifth, etc.)= $\frac{1}{5}+\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}+\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{1}{5}+\cdots = \frac{1}{5}\left(1+\frac{4^2}{5^2}+\frac{4^4}{5^4}+\dots\right) = \frac{1}{5}\cdot\frac{1}{1-16/25} = \frac{1}{5}\cdot\frac{25}{9} = \frac{5}{9}$.

Coin Problem. First, use von Neumann's trick to obtain an unbiased string of 0 and 1 as if from a unbiased coin. Then read the first four digits. If 0000, output **1**. If 0001, 0010, 0011, output **2**. If 0100, 0101, 0110, 0111, 1000, output **3**. If either of the remaining seven outcomes, consider the next four digits until the desired pattern is found. Check that the resulting probabilities of **1**, **2**, **3** are as desired.