MIDTERM 2 (18.05, SPRING 2003)

1) Let X has a Poisson distribution so that $P(X=2) = 12\,P(X=4)$. Compute $P(X \geq 4 \mid X \geq 2)$ and $P(X=17)$.

2) Let X be $\chi^2(7)$. Write a close formula for the p.d.f. (the answer should not contain gamma function)[1]. Let $Y = X_1 + X_2 + \cdots + X_{16}$, where X_i are mutually independent and $\chi^2(7)$. Compute the mean and variance of Y.

3) Suppose X is $N(8, 64)$. Use the table for $N(0,1)$ to estimate:

a) $P(X \leq 32)$

b) $P(X \geq -4)$

c) $P(0 \leq X \leq 12)$

d) $P(X \geq 12 \mid X \geq 32)$

4) Suppose X is $\chi^2(1)$. Use the table for $N(0,1)$ to estimate:

a) $P(X \leq 1.44)$

b) $P(X \leq 4.41 \mid X \geq 1.44)$

c) Compare a) with the estimate given by the Chebyshev bound.

5) University admissions sends every year 10,000 acceptance to random HS seniors who have SAT scores between 600 and 750. Of these, a random 1000 seniors accept the offer. University then offers financial aid to the top 20% of those accepted, based solely on their SAT scores. Assuming SAT scores are $N(500, 100^2)$, find the lowest SAT for a student to be eligible to receive financial aid. Suppose the financial aid is $10K$ for all eligible students. Find the amount the university spends on those seniors in the incoming class with the SAT scores ≥ 700.

6) Suppose there are 60 people at a party. Two people are "surprised" if their birthdays differ by at most two days. Assuming there are 365 days in a year and birthdays are uniformly distributed, compute the expected number of "surprised" pairs of people.

Say, three people are "shocked" if they have consecutive birthdays. Under the same assumptions, estimate the expected number of "shocked" triples of people.

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[1] *Hint:* use the formula for $\Gamma(\frac{1}{2})$.