MIDTERM 1 (18.05, SPRING 2003)

1) Assume A, B, C are mutually independent events, $P(A) = 1/2$, $P(B) = 2/3$, $P(C) = 3/5$. Compute:

a) $P(A \cup B)$

b) $P(A \cap B \mid C)$

c) $P((\overline{A} \cap \overline{B}) \cup C)$

2) A teacher tabulates the number of books of students in class. Turns out, there are $2, 6, 6, 10, 6$ students who have $0, 1, 2, 3, 4$ books, respectively. If X is the number of books a random student has, compute $E(X)$ and $Var(X)$.

3) Let $f(x) = c/x$ for $1 \leq x \leq 10$ and $f(x) = 0$ otherwise.

a) Find c such that f is a probability density function.

b) Sketch $f(x)$ and the distribution function $F(x)$

c) Compute $E(X)$ and $Var(x)$

4) The urn has 12 balls, of which 3 are green and all other are white. Two players take turns removing balls from the urn (without replacement) until no balls are left. Compute

a) Probability that the first player ends up with all 3 green balls

b) Probability that both players end up with the same number of white balls.

5) A town has a basketball and a football team which play in tournaments once a week. Assume the basketball team wins with probability $1/3$ and football team wins with probability $1/4$ and the teams performance is independent of each other. Compute the expected wait time until both teams win on the *same* week.

⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆