HOMEWORK 6 (18.05, SPRING 2003)

Solve: Problems below.

H6-1) There are 24 white balls and 6 red balls in the urn. Five players take turns collecting them, until each has 6 balls.

a) Compute the expected number of red balls the first and the second player have (combined).

b) Compute the expected difference in the number of red balls of the first and the second player.

H6-2) At a party of n people, random two know each other with probability p and these probabilities are mutually independent. Compute the expected number of ways these n people can stand in line so that each person knows the next one in line (the last one has nobody behind, so must know only the previous person). Make reasonable assumptions on the number of people and the size of the party and decide whether such line is likely to exist at all.

H6-3) Suppose n people have their birthdays uniformly distributed. Compute the expected number of triples of people who share the same birthday. What is this (roughly) this expectation for $n = 60$ people? How large should n be so that this expectation is ≥ 1?

H6-4) A statistician claims that in a country XYZ the percentage of people who make $\geq 300K$ is about 40, of those making $\geq 900K$ is about 15, and of those making $\geq 3,000K$ is about 1. Do these facts contradict each other? Which of these facts contradict an (established independently) fact that the average annual income is $100K$. Defend statistician's work by computing the percentage of error in each case.

H6-5) The average March temperature in town X is $50°$. Assume March has 4 weeks. Suppose the average weekly degree increases by $5°$ from week to week. Use Markov inequality to bound from above the probability that the average temperature in the first week of March was $\geq 80°$.

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This Homework is due Monday Mar 31 at 10:55 am.
Hand in after class.

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