MIDTERM 1 (MATH 61, SPRING 2017)

Your Name: ______________________________

UCLA id: ______________________________

Math 61 Section: ________

Date: ______________

The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no web access. You MUST write your name and UCLA id.
Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught violating the rules get automatic 10% score deduction.

Points:

1	
2	
3	
4	
5	

____

Total: **(out of 100)**

Problem 1. (20 points)

Compute the probability that 4-subset A of $\{1, 2, \ldots, 10\}$ satisfies:

$a)$ A has no odd numbers,

$b)$ A has at least one number ≤ 3,

$c)$ A contains 1 but not 7.

$d)$ the smallest number in A is divisible by 3

Problem 2. (20 points)

Let $X = \mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of all non-negative integers. For each of the following functions $f : X \to X$ decide whether they are injective, surjective, bijective:

a) $f(x) = x + 1$

b) $f(x) = x^2 - 1$

c) $f(x) = 2x$

d) $f(x) = (x^2 + 2x)/(x + 2)$

Problem 3. (15 points)

Let $a_n = 1111 \cdots 1$ (n ones). Suppose a_k is divisible by 97. Use induction to show that $a_{k \cdot n} = 0 \bmod 97$, for all $n \geq 1$.

Problem 4. (15 points)

Find closed formulas for the following sequences :

a) $4, 4, 6, 8, 12, 18, 28, 44, 70, 112, \ldots$

b) $a_1 = 1,\ a_{n+1} = a_n \cdot \binom{n+1}{2}$

c) $a_1 = 1,\ a_2 = 1,\ a_{n+1} = a_{n-1} - a_n$ for $n \geq 2$.

Note: you can express a_n in terms of Fibonacci numbers F_n.

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T **F** (1) The number of functions from $\{A, B, C, D\}$ to $\{1, 2, 3\}$ is equal to 4^3.

T **F** (2) The sequence $1, 3/2, 5/3, 7/6, 9/8, \ldots$ is increasing.

T **F** (3) The sequence $-1, -2, -3, -4, \ldots$ is non-increasing.

T **F** (4) There are 4 anagrams of the word MAMA.

T **F** (5) There are infinitely many Fibonacci numbers which are divisible by 3.

T **F** (6) The number of permutations of $\{1, 2, 3, 4, 5\}$ is smaller than 123.

T **F** (7) The number of 3-permutations of $\{1, 2, 3, 4, 5, 6\}$ is equal to $\binom{6}{3}$.

T **F** (8) The number of 3-subsets of $\{1, 2, 3, 4\}$ is equal to 4.

T **F** (9) The number of permutations of $\{1, 2, \ldots, n\}$ which have n preceding $n - 1$ (not necessarily immediately) is equal to $n!/2$

T **F** (10) For every $A, B \subset \{1, 2, \ldots, 12\}$ we have $|A \cap B| < |A \cup B|$.

T **F** (11) For all $n \geq 1$, we have

$$\binom{2n}{0} + \binom{2n}{2} + \binom{2n}{4} + \ldots + \binom{2n}{2n} = 2^{2n-1}.$$

T **F** (12) The number of grid walks from $(0, 0)$ to $(10, 10)$ going through $(3, 7)$ is equal to $\binom{10}{3}^2$.

T **F** (13) The number of grid walks from $(0, 0)$ to $(10, 10)$ avoiding $(10, 0)$ and $(0, 10)$ is equal to $\frac{1}{2}\binom{20}{10}$.

T **F** (14) The number of anagrams of MISSISSIPPI which begin with M is greater than the number of anagrams which begin with S.

T **F** (15) The following parabolas are drawn in the plane:

$$y = x^2 - n^2 x - n^3, \qquad n = 1, \ldots, 12.$$

Then the regions of the plane separated by these parabolas can be colored with two colors in such a way that no two adjacent regions have the same color.