HOMEWORK 2 (MATH 61, SPRING 2017)

Read: **RJ**, Sec. 3.3, 3.4, 6.1, 6.8.

Solve: **RJ**, Sec. 3.3 Ex 25, 26, 28, 29, Sec. 3.4 Ex 6, 8, 10, 11, 13, 14, Sec 6.1 Ex 6, 8, 42, 43, 88, 90, 91, Sec 6.8 Ex 3, 6, 8.

I. Let R be a relation on $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$ defined as follows. Decide whether or not it's a reflexive, symmetric, transitive and equivalence relation. If R is an equivalence relation, describe the equivalence classes.

Below $x, y \in \mathbb{Z}$ and xRy if and only if

a) $(x+y)^2 = \pm 1 \bmod 4$
b) $(x-y)^2 = 0 \bmod 4$
c) $x + 2y = 0 \bmod 5$
d) $x^2 - 4y^2 = 0 \bmod 5$
e) $x^2 + 2x + 1 = y^2 + 2y + 1$
f) $x^2 + 6x + 5 = y^2 + 6y + 5$
g) $x^2 + y^2 = 25$
h) $x^2 - y^2 = 6$

II. Let R be a relation on the plane $\mathbb{R}^2 = \{x = (a, b)\}$ defined as follows. Decide whether or not it's a reflexive, symmetric, transitive and equivalence relation. If R is an equivalence relation, describe the equivalence classes.

Below $x = (a, b) \in \mathbb{R}^2$, $y = (c, d) \in \mathbb{R}^2$ and xRy if and only if

a) $|Ox| = |Oy|$
b) $|Ox|^2 - |Oy|^2 = 1$
c) $|Ox| + |Oy| = |xy|$
d) $a - b = c - d$
e) $a^2 + b = c^2 + d$
f) $ab = cd$
g) $a = c = 3$, $b = d = 2$
h) $a^3 + b^3 = c^3 + d^3$

III. Use variations on the Pigeonhole Principle to prove the following results:

a) Let $A \subset \mathbb{N}$ be a set of 100 integers. Prove that there is a subset $X \subseteq A$ such that the sum of all elements in X is divisible by 100.

b) There are 900 points in the circle of radius 1. Prove that at least two of them are at distance < 0.1.

c) A music club has 33 members and total sum of their ages is 430. Prove that 20 oldest club members have sum of their ages greater 260.

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This Homework is due Wednesday April 19, at 10:59:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Write the answers in inc and box them. Remember that in the proof questions, you also need to provide an explanation exhibiting your logic. In other questions, just the answer suffices.

P.S. Each item above has the same weight.