

HOMEWORK 1 (MATH 61, SPRING 2017)

Read: **RJ**, sections 2.2, 2.4, 3.2.

Solve: **RJ**, Sec. 2.2 Ex 28, 29, Sec. 2.4 Ex 2, 3, 6, 9, Sec 3.2 Ex 7, 9, 13, 14.

I. Prove by induction:

$$\begin{aligned} a) \quad & \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n} \\ b) \quad & 1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2 \end{aligned}$$

II. Let $a_1 = a_2 = 1$, $a_{n+1} = a_n - a_{n-1}$. Prove that $-3 \leq a_n \leq 3$ for all n .

III. Find closed formulas for elements in the following sequences:

- a) 1, 3, 5, 7, 9, 11, ...
- b) 1, -4, 10, -20, 35, -56, ...
- c) 1, 3/2, 6, 3/24, 120, 3/720, ...
- d) 1/4, -4/9, 9/16, -16/25, 25/36, ...
- e) 1, 1/5, 1/21, 1/85, 1/341, 1/1365, ...

IV. For the following sequences, Compute the first 5 elements. Then decide whether they *are* or *are not* increasing, decreasing, nonincreasing, and nondecreasing.

$$a_n = n - 3^n$$

$$b_n = n + \frac{1}{n}$$

$$c_n = 3 - \frac{1}{n}$$

$$d_n = \frac{(-1)^n}{n^2}$$

$$e_n = \frac{2^n + 3^n}{13n^2}$$

This Homework is due Wednesday April 12, at 10:59:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your **name**, your **UCLA id number**, and your **section name** (A,...,F) in the beginning and your collaborators' names at the front page. Write the answers in **ink** and **box** them. Remember that in the proof questions, you also need to provide an explanation exhibiting your logic. In other questions, just the answer suffices.

P.S. Each item above has the same weight.