Homework 8

11.4 $\int_c^\infty \int_c^\infty (x+y)^{-3}\, dx\, dy$

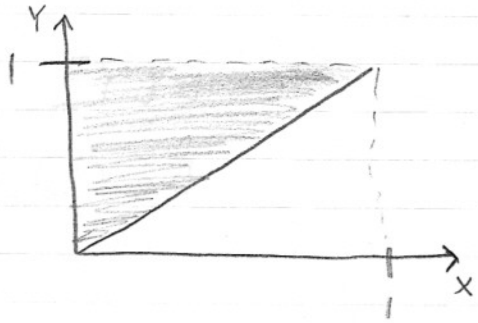
$= \int_c^\infty \left[\frac{(x+y)^{-2}}{-2}\right]_c^\infty dy = \int_c^\infty \frac{(c+y)^{-2}}{2} dy$

$= \left[-\frac{(c+y)^{-1}}{2}\right]_c^\infty = \frac{1}{2(2c)}$

$\frac{1}{4c} = 1 \Rightarrow \boxed{c = \frac{1}{4}}$

$\boxed{P(X>a, Y>b) = \int_a^\infty \int_b^\infty \frac{1}{(x+y)^3}\, dy\, dx}$

I. $c\int_0^1 \int_x^1 y^2 - x^2\, dy\, dx$



$= c\int_0^1 \left[\frac{y^3}{3} - x^2 y\right]_x^1 dx$

$= c\int_0^1 \frac{1}{3} - x^2 + \frac{2x^3}{3}$

$= c\left[\frac{x}{3} - \frac{x^3}{3} + \frac{2x^4}{12}\right]_0^1$

$= c\left[\frac{1}{3} - \frac{1}{3} + \frac{2}{12}\right] \Rightarrow c = 6$

$E(X) = \int_0^1 \int_x^1 xy^2 - x^3\, dy\, dx$

$E(X^2) = \int_0^1 \int_x^1 x^2y^2 - x^4\, dy\, dx$

$E(Y) = \int_0^1 \int_0^y y^3 - x^2 y\, dx\, dy$

$E(Y^2) = \int_0^1 \int_0^y y^4 - x^2 y^2\, dx\, dy$

correlation coefficient $= \frac{E[XY] - E[X]E[Y]}{\sqrt{Var[X]}\sqrt{Var[Y]}}$

II. For $\angle aOb$ to be obtuse, a must be on the opposite side of the circle as b.

This probability is equal to $\frac{1}{2}$