MIDTERM PRACTICE PROBLEMS
(MATH 4653, FALL 2008)

1. $P(x=0)=1/2^9$, $P(x=1)=9/2^9$, $P(x<y)=P(x>y)=1/2$, $P(x^2+y^2=10)=P(x=1,y=3)+P(x=3,y=1)=0+0=0$.

2. a) $P(a_1+a_2=8)=6\cdot 6!/8!=3/28$, $P(a_1\cdot a_2=2)=2/56=1/28$,
b) $P(a_1+a_2=8 \mid a_1\cdot a_2=2)=0$, $P(a_1+a_2=8 \mid a_3\cdot a_4=2)=2\cdot 4!/6!$.

3. $P(A)=.50$, $P(B)=.90$, $P(C)=.15$, $P(A\cap B)=.45$, $P(A\cap C)=.15$, $P(B\cap C)=.12$. Thus A,B are independent, while A,C are not, B,C are not.

4. Clearly only numbers 3, 13, 23 have digit 3. Thus $3p+(30-3)2p=1$, $p=1/57$. $P(X<5)=2p+2p+2p+p+2p=9/57=3/19$, $P(X<10)=2p+2p+2p+p+2p+2p+2p+2p+2p=19/57=1/3$, $E[X]=1\cdot 2p+2\cdot 2p+3\cdot p+\ldots=(1+2+\ldots+29)\cdot 2p-(3+13+23)p=(29\cdot 30-39)p=831/57$.

5. $P(A)=x$, $P(A\cap B)=P(B\mid A)\cdot P(A)=x/3$, $P(B)=P(B\cap A)/P(A\mid B)=5x/3$. Now $0.7=P(A\cup B)=P(A)+P(B)-P(A\cap B)=x+5x/3-x/3=7x/3$, $x=0.3$.

6.

a. $E[X]=1/2+2/3+3/6=5/3$, $Var(X)=1/2+4/3+9/6-(5/3)^2=10/3-25/9=5/9$.

b. $E[Y]=2-3\cdot 5/3=-3$, $Var(Y)$, where $Y=3^2\cdot 5/9=5$.

7. $p=P(\text{stop})=1/8$, X is Geo(p),

a. $P(2\le X\le 5)=(1-P(X>5))-P(X=1)=1-(1-p)^5-p$.

b. $E[X]=8$ and $Var(X)=(1-1/8)/(1/8^2)=56$.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

8. True or False? (give only answers, no calculations are necessary)

a. (FALSE) In the coupon collector's problem with 50 coupons, the expected time to collect the first 25 coupons is more than half the expected time to collect all 50 coupons.

Later coupons take longer.

b. (FALSE) For all events A, B we have $P(A \cup B) \geq 1 - P(A \cap B)$.

Take $P(A) = P(B) = 0.1$, $P(A \cap B) = 0$. Then $P(A \cup B) = 0.2 < 1 - P(A \cap B) = 1$.

c. (TRUE) For every X with Poi(λ) distribution, we have $P(X \geq 4\lambda) \leq 0.25$.

This is a special case of the Markov inequality.

d. (FALSE) Every two Bernoulli distributions are independent.

Take X and $1 - X$, where X is any Bernoulli

e. (TRUE) Every Bernoulli distribution is also a Binomial distribution.

Yes. $Ber(p) = Bin(1, p)$.

f. (TRUE) The variance of a geometric distribution is smaller than the square of the mean.

$(1 - p)/p^2 < (1/p)^2$.

g. (FALSE) In the princess marriage problem with 100 princes the odds of choosing the best prince is the second strategy of following two strategies is better:

1. skip the first prince and choose the first prince who is better than at least one of the previous princes.
2. skip the first 50 and choose the first prince who is better than at least one of the previous princes.