HOMEWORK 7 (4653, FALL 2008)

Read: *Tijms*, pages $284 - 299$, $313 - 321$.

Solve: Problems 10.2, 10.6, 10.10, 10.14, 10.16, and the following problems:

I. (20 points)
Let A be a region in the plane defined by the following inequalities:

$$A = \{0 \le x,\ 1 \le x + y \le 2,\ 0 \le y \le 1\}.$$

Denote by (X, Y) the coordinates of a random point in A, i.e. chosen from a uniform distribution on A.

    $a)$ Find the marginal probabilities for X and Y.

    $b)$ Find the expectations $E[X]$ and $E[Y]$.

    $c)$ Find the variances $Var(X)$ and $Var(Y)$.

    $d)$ Find the probabilities $P(X \le 1)$, $P(X < Y)$, $P(X^2 < Y)$.

II. (20 points)
Let B be a region in the plane defined by the following inequalities:

$$B = \{0 \le x \le 1,\ 0 \le y \le 1,\ x \ge y,\ y \ge x^2\}.$$

Denote by (X, Y) the coordinates of a random point in B, i.e. chosen from a uniform distribution on B.

    $a)$ Find the marginal probabilities for X and Y.

    $b)$ Find the expectations $E[X]$ and $E[Y]$.

    $c)$ Find the variances $Var(X)$ and $Var(Y)$.

    $d)$ Find the probabilities $P(X \le 0.5)$, $P(Y \le 0.5)$, $P(X + Y \le 1)$.

III. (20 points) Let X be a c.r.v. with p.d.f. $f(x) = c/x^2$ for $1 \le x \le 2$ and $f(x) = 0$ otherwise.

$a)$ Find c.

$b)$ Compute the c.p.f. $F(a)$.

$c)$ Find $P(X \le 1.3)$, $P(X^2 \ge 2)$, $P(1.1 \le X \le 1.6)$.

$d)$ Find $E[X]$ and $Var(X)$.

IV. (20 points) Let Y be a c.r.v. with p.d.f. $f(x) = c|y - 2|$ for $-3 \le x \le 3$, $f(y) = c(y + 4)$ for $-4 \le y \le -3$ $f(y) = c(4 - y)$ for $3 \le y \le 4$, and $f(x) = 0$ otherwise.

a) Find c.

b) Compute the c.p.f. $F(b) = P(Y \leq b)$.

c) Find $P(Y \leq 1)$, $P(Y^2 \geq 2)$, $P(-1.3 \leq Y \leq 3.1)$.

d) Find $E[Y]$ and $Var(Y)$.

V. (10 points) Let k and ℓ be two random digits between 0 and 9. Denote by $X = k + \ell$ and $Y = k \cdot \ell$. Compute $\rho(X, Y)$.

VI. (10 points) In a small country town there is only one school which teaches 40 students in each grade from 1 to 12. The enclosed table of grades gives (rounded) GPA of all students in each class. The town officials are afraid that older students seem to have better GPA, and claims that teachers slightly inflate the grades to prepare them for college. The school officials reject this speculation. A decision is made to call a statistician to compute the correlation coefficient $\rho(X, Y)$, where X is the grade and Y is the GPA. If $\rho > 10\%$, the principal agreed to resign. Make a computation to decide if the school needs to start looking for a new principal.

VII. (15 points) Look at the US Census Bureau table H13, under "*Historical Income Tables - Households*":

  http://www.census.gov/hhes/www/income/histinc/h13.html

Consider only mean income in 2007 dollars. Assume that students with at most 9 years studied for an average of 8 years in school, students with no HS diploma for 10 years, student with HS diploma for 12 years, students with some college for 13 years, students with associate degree for 14 years, students with bachelor's for 16 years, with master's degree for 18 years, and with professional degree for 20 years. Find a correlation between income and years of education for each of the years: 1991, 2000 and 2007 (make sure to account on the number of households in the first column). Decide whether the effect of education on income becomes greater or less.

VIII. (10 points) Look at the US Census Bureau table H2, under "*Historical Income Tables - Households*":

  http://www.census.gov/hhes/www/income/histinc/h02AR.html

Make a smaller table of Share of Aggregate Income percentages for the years 1967, 1972, 1977, ... 2007 (every 5 years). Compute the correlation coefficient between the years and the quintile percentages. Some social scientists are saying that "rich get richer", i.e. the better paid people's share of overall wealth is actually increasing with time. Does your finding confirm or dispute this assertion (use 10% correlation as a threshold for accepting/rejecting).

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This Homework is due Tuesday December 2 at 4:40 pm. (right before the class). You MUST box all the answers. Remember that the answers are not enough, you also need to provide all intermediate calculations exhibiting your logic. However, no written explanations are necessary unless the problem asks for them.