HOME ASSIGNMENT 1   (MATH 206A, FALL 2020)

I. Find asymptotics for the number of posets of height two (up to isomorphism). Same for posets of width two. Compare the asymptotics.

II. Let P_n be a poset of subspaces of $\mathbb{F}_2^n$ by inclusion. Compute the number of maximal chains in P_n. Same for the total number of chains in P_n. Give asymptotics if you can't give an exact formula.

III. Let n be even. Give asymptotics for the number of antichains in P_n.

IV. Let Q_n be the 2-dim poset of points $\{(i,j), 1 \leq i, j \leq n\} \subset \mathbb{N}^2$. Compute the number of maximal chains and antichains in Q_n. Give asymptotics for the total number of chains and antichains in Q_n. Compare the asymptotics.

V. For posets $P = (X, \prec)$ and $Q = (Y, \prec')$ define the *sum* $P + Q$ and the *product* $P \cdot Q$ on the same set $X \cup Y$, as in the figure. Define also a 4-element poset Z as in the figure. Prove that a poset A does not contain Z as an *induced subposet* (meaning it has no additional relations), if and only if A can be obtained from a single-element poset using the sum and product operations.
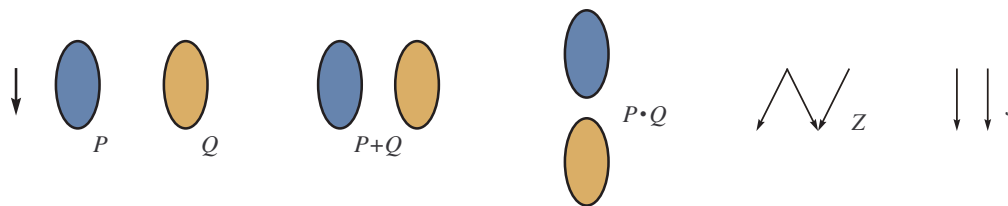
VI. Use the previous problem to prove that the number of non-isomorphic posets on n elements which have no induced Z and J as in the figure, is the Catalan number C_n.

VII. Consider all *point-and-line configurations* obtained by taking n (labeled) points in $\mathbb{F}_q^2$ and taking all lines which contain at least 2 of these points. For a poset P of height two with n minimal elements, denote by $f_P(q)$ the number of such configurations isomorphic to P. Prove or disprove: $f_P(q)$ is a polynomial in q for all P, where $q > n^2$ is prime.

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This Homework is due Monday Oct 19, at 1:59 pm (right before class). The solutions must be uploaded to the Gradescope. Please read the collaboration policy on the course web page. Feel free to look for collaborators on the CCLE chat. Do not discuss there any solution ideas, and only use the chat to clarify the statements of the problems. Make sure you write your **name**, your **UCLA id number**, and your collaborators' names at the front page.

P.S. Each problem above has the same weight.