

HOMEWORK 5 (MATH 184, WINTER 2017)

Read: Bona (Second ed.), sections 2.7, 3.7, 3.3, 3.4, 4.1, 4.4, 4.5.1, 4.7, Stanley's survey pp. 1-2.

Solve: 22, 23, 25 in §3.10, 22, 27, 44 in §4.10 and the following problems:

- I. Prove that Catalan number C_n is odd if and only if $n = 2^k - 1$ for some $k \geq 1$.
- II. Prove that the number $a(2m+1)$ of alternating permutations $\sigma \in S_{2m+1}$ is divisible by 2^m .
- III. Let $b(n, k)$ be the number of permutations with k cycles and no fixed points. Find recurrence relations for $b(n, k)$. Let

$$B(t, z) = \sum_{n=1}^{\infty} \sum_{k=1} b(n, k) t^n z^k$$

Write explicitly partial differential equation that $B(t, z)$ satisfies.

This Homework is due Friday March 17, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.