**HOMEWORK 4 (MATH 184, WINTER 2017)**

**Read:** **Bona** (Second ed.), sections 4.1-3.

**Solve:** The following problems:

**I.** Use generating functions to find the expectation and the variance of the number of inversions in random $\sigma \in S_n$.

**II.** Find the exponential g.f. for the number of permutations with cycle lengths divisible by 3. Use this to compute a closed formula for the number of such permutations.

**III.** For the following functions decide which pairs satisfy $f(n) = O\big(g(n)\big)$, $f(n) = o\big(g(n)\big)$, $f(n) \sim g(n)$ and $f(n) = \Theta\big(g(n)\big)$

$n^{\log n}$, $n^9$, $n^n$, $\exp\big(n^2/(n-1)\big)$, $(2n-1)!!$, $\exp\big((\log n^2)^2\big)$, $n^{\sqrt{n}}$, $\log(n!)$, $n^{n^2}$, $e^n(2+\sin n)$

**IV.** Give an inductive proof that the number of increasing binary trees with $n$ vertices is $n!$

**V.** For the number $a_n$ of permutations with cycles of lengths 1 and 2, prove the recurrence formula $a_n = a_{n-1} + (n-1)a_{n-2}$. Check that the summation formula in class (or in Bona, §4.3) satisfies this recurrence. Prove that $n! = o(a_n^2)$.

⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆⋆

This Homework is due Wednesday March 8, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.