HOMEWORK 3 (MATH 184, WINTER 2016)

Read: **MB**, Sections 2.3, 3.1, 3.2.1, 3.3.1, 3.3, 4.4.

Solve: Supplementary exercises
19, 20, 21, 22, 23 in §2.10,
3, 4, 6, 7, 11, 12 in §3.10,
1, 3, 6, 8, 13, 14 in §4.10,
and the following problems.

I. Let $Z = \mathrm{inv}(\sigma)$ be the number of inversions of random permutation $\sigma \in S_n$. Recall $E[Z] = n(n-1)/4$ we proved in class. Find $Var(Z)$.

II. Prove the following identity:

$$\prod_{i=1}^{\infty} \frac{1}{1-t^i} \;=\; 1 + \sum_{k=1}^{\infty} \frac{t^{k^2}}{(1-t)^2(1-t^2)^2\cdots(1-t^k)^2}$$

Hont: Use Durfee square defined on p.99.

III. Let $p_0(n)$ be the number of partitions of n with an even number of even parts. Similarly, let $p_1(n)$ be be the number of partitions of n with an odd number of even parts. Prove that $p_0(n) - p_1(n)$ is equal to the number of partitions of n into distinct odd parts.

IV. Prove that there are infinitely many n for which $p(n)$ is odd.

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This Homework is due Wednesday February 17, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Remember to give a full proof.

P.S. Each item in the problems above has the same weight.