HOMEWORK 1 (MATH 184, WINTER 2016)

Read: **MB**, Chapter 1 and sections 2.1, 2.2.

Solve: Supplementary exercises 3,4,5,6 in §1.10 (p. 49) and the following problems.

I. Throughout the problem, assume $n = 12$ and $k = 5$. Compute the number of k-subsets A of $[n]$, such that:
  - $a)$ A contains 1 and n
  - $b)$ A contains 1 but not n
  - $c)$ A contains 1 or n
  - $d)$ A contains an integer ≤ 4
  - $e)$ A contains at least one integer ≤ 3 and is missing at least one integer ≤ 6
  - $f)$ A contains at least two integers ≤ 6
  - $g)$ A contains only odd integers
  - $h)$ A contains exactly one even integer
  - $i)$ the product of numbers in A is 240
  - $j)$ the product of numbers in A is ≤ 150
  - $x)$ no two numbers in A are consecutive
  - $y)$ the smallest number in A is the largest number in A minus 10
  - $z)$ the sum of squares of the numbers in A is equal to 370.

II. Find a closed form for the following summation:

$$\sum_{k=0}^{n} k^4 \binom{n}{k}$$

III. Prove that $\binom{2^k}{\ell}$ is even for all $k \geq 1$, $1 \leq \ell \leq 2^k - 1$. Conclude from here that $\binom{2^k-1}{\ell}$ is odd, for all $0 \leq \ell \leq 2^k - 1$.

IV. A subset S with 101 numbers was chosen uniformly at random from $[1001]$. Use Stirling's formula to approximate the following probabilities:
  - $a)$ that the median number in S is 501
  - $b)$ that the median number in S is 551
  - $c)$ that the median number in S is 601
  - $d)$ that the 26th number in S is 251 and 76th number in S is 751
  - $e)$ that the 26th number in S is 301
  - $f)$ that S has no consecutive numbers

V. Let S be a random k-subset of $[n]$. Assume that both k and n are even. Compute:
  - $a)$ the expected number of even numbers in S,
  - $b)$ the expected number of consecutive pairs of numbers in S (i.e. number of $i \in S$ such that $i + 1$ is also in S),
  - $c)$ the expected sum of numbers in S,
  - $d)$ the expected sum of squares of numbers in S.

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This Homework is due Wednesday January 20, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Remember to give a full proof.

P.S. Each item in the problems above has the same weight.