

HOME ASSIGNMENT 4 (MATH 184, SPRING 2022)

Read: Bona (Second ed.), sections 3.4, 4.3, 5.5

Solve: Exc 22 in §3.10, 21, 22, 23, 28 in §4.10, 30 in §5.10, and the following problems:

I. Recall the Entringer numbers $a_{n,k}$ for the number of alternating permutations which start with k . Find a recurrence relation for $a_{n,k}$ to justify them forming a Pascal-type triangle as shown in class.

II. Let T_n be the number of labeled trees on n vertices which increase away from the root at 1, i.e. for every $1 < i < j \leq n$, the shortest path from i to 1 cannot contain j . For example, $T_3 = 2$. Prove that $T_n = (n - 1)!$.

III. Let L_n be number of labeled trees on n vertices which increase away from the root at 1, such that every vertex has degree at most 3 and root has degree at most two. For example, $L_4 = 5$. Prove that $L_n = a_n$ is the number of alternating permutations.

IV. Let b_n be the number of permutations of order 6, i.e. number of $\sigma \in S_n$ such that $\sigma^6 = 1$. Find a polynomial recurrence for b_n . Find a closed form Exponential GF for b_n .

This HA is due Friday June 3, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.