HOME ASSIGNMENT 2   (MATH 184, SPRING 2022)

Read: **Bona** (Second ed.), sections 2.3, 3.1, 3.2.1, 3.3.1, 3.4.1.

Solve: Exc 19, 20, 21, 22, 23, 26 in §2.10,   3, 4, 6, 7, 12 30 in §3.10, and the following problems:

I. Let $p_\circ(n)$ denote the number of partitions of n into parts $\{1, 5, 10, 25, 50, 100\}$.
   $a)$ Prove that $p_\circ = \Theta(n^5)$
   $b)$ Prove that $p_\circ \sim C \cdot n^5$ for some $C > 0$
   $c)$ Find C.

II. Let $k = 20$, $n = 50$. Denote by S a random k-subset of $[n]$.
   $a)$ Let $X := \big|S \cap [20]\big|$. We found $\mathbb{E}[X]$ in class. Find $\mathrm{Var}(X)$.
   $b)$ Let $Y := \sum_{m \in S} m^2$. Find $\mathbb{E}[Y]$ and $\mathrm{Var}(Y)$.
   $c)$ Let $Z := \sum_{m \in S \cap [20]} m^2$. Find $\mathbb{E}[Z]$ and $\mathrm{Var}(Z)$.

III. Let X be a discrete RV which takes values in $\mathbb{N}$, and let

$$A(t) = \sum_{n=0}^{\infty} P[X = n] \cdot t^n$$

   $a)$ Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$ if

$$A(t) = \prod_{i=1}^{n} \left(\frac{i}{n} + t\, \frac{n-i}{n} \right)$$

   $b)$ Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$ if

$$A(t) = \frac{1}{n!} \prod_{i=1}^{n} \big(1 + (i-1)t\big)$$

   $c)$ Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$ if

$$A(t) = \frac{1}{\binom{n}{k}} \frac{(n!)_t}{(k!)_t\,(n-k)!_t}$$

   where $(k!)_t := (1)_t \cdot (2)_t \cdots (k)_t$ and $(k)_t = (t^k - 1)/(t - 1)$, so in particular we have $(k)_1 = k$ and $(k!)_1 = k!$.

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   This Homework is due Wednesday April 27, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.