MIDTERM (MATH 182, SUMMER 2022)

Your Name: ____

UCLA id: ____

The rules:
Open book, open notes, open internet.
You MUST write your in your own handwriting the pledge on the second page.
You MUST simplify completely and BOX all answers. Except for the last problem,
you MUST write out your logical reasoning and/or calculation steps in full.
You MUST work on this midterm for exactly 50 minutes within the 24 hour window.

Please write in your own handwriting the pledge below:

This work was performed solely by the me [your name, UCLA id], in accordance with the rules above. No part of this midterm was shared with anyone in or outside of the class.

Problem 1. (20 points, 10 points each)

Consider an array $(2, 6, 19, 23, 1, 4, 5, 20)$.

$a)$ Show the steps of the `mergesort`.

$b)$ Use part $a)$ to compute the number of inversions in the array.

Problem 2. (20 points)

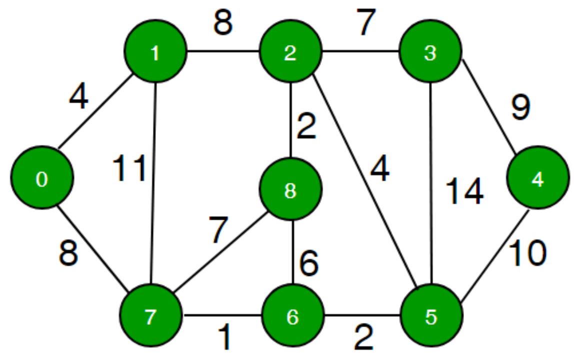
Consider the following five functions:

$$f(n) = n^2, \quad g(n) = n^n, \quad h(n) = 2^{n^2}, \quad u(n) = 2^{n^2 - n\log n}, \quad v(n) = 2^{n^2 - n + 1}$$

Find all pairs of these functions for which one = o(another). Here we have the "little-o" notation.

Problem 3. (15 points)

Use Dijkstra's algorithm to find the minimum weight path from 0 to 4 in the figure below. Is such path unique?



Problem 4. (15 points)
Consider a complete graph on 6 vertices with labels given by cities, and weights given by distances as in the figure below. For example, the distance between Exeter and Plymouth is 44. Use Kruskal's algorithm to compute the minimal weight spanning tree. Is such tree unique?

	Barnstaple	Bristol	Exeter	Penzance	Plymouth
Bristol	100				
Exeter	55	84			
Penzance	108	194	110		
Plymouth	67	125	44	77	
Taunton	50	51	34	144	75

Problem 5. (30 points, 2 points each). **True or False?**

Fill in the circles with correct answers. No reasoning/calculations will be taken into account.

T ◯ **F** ◯ 1. Bubble sorting cannot be used to compute the number of inversions

T ◯ **F** ◯ 2. In weighted graphs with negative cycles, the minimal weight path might not exist

T ◯ **F** ◯ 3. In the BFS of the graph in Problem 3, if you start at vertex 0, then the last vertex to explore is 5

T ◯ **F** ◯ 4. In the DFS of the graph in Problem 3, if you start at vertex 0, then the last vertex to explore is 8

T ◯ **F** ◯ 5. The fastest algorithm for computing the minimal distance between two points has time complexity $O(n^{3/2})$

T ◯ **F** ◯ 6. The Karatsuba integer multiplication algorithm of two integers written in binary with n digits each, has time complexity $o(n^2)$

T ◯ **F** ◯ 7. Subtraction of two integers written in binary with n digits each, has time complexity $O(n)$

T ◯ **F** ◯ 8. All graphs which do not have cycles of length 3 are bipartite

T ◯ **F** ◯ 9. The array $(n, n-1, \dots, 1)$ has $O(n^{2.59} \log n)$ inversions

T ◯ **F** ◯ 10. Repeated squaring can be used to compute Fibonacci numbers

T ◯ **F** ◯ 11. Knapsack problem can be solved (perhaps, slowly) by polynomial multiplication

T ◯ **F** ◯ 12. Kruskal's algorithm belongs to the category "Divide and conquer algorithms"

T ◯ **F** ◯ 13. Karatsuba multiplication belongs to the category "Greedy algorithms"

T ◯ **F** ◯ 14. The largest independent set in a graph on n vertices can be found in $O(n \log n)$ time by a DFS algorithm

T ◯ **F** ◯ 15. Master's theorem is used to bound time complexity in "Divide and conquer algorithms"