MIDTERM (MATH 180, WITER 2021)

Your Name: ____

UCLA id: ____

Date: ______________________________________

The rules:
Open book, open notes, open internet.
You MUST write your in your own handwriting the pledge on the second page.
You MUST simplify completely and BOX all answers. Except for the last problem, you MUST write out your logical reasoning and/or proof in full.
You MUST work on this midterm for exactly 50 minutes within the 24 hour window.

Points:

1	
2	
3	
4	
5	

____

Total: **(out of 100)**

Please write in your own handwriting the pledge below:

This work was performed solely by the me [your name, UCLA id], in accordance with the rules above. No part of this midterm was shared with anyone in or outside of the class.

Problem 1. (18 points, 3 points each part)
In each case, compute the number of subgraphs of G isomorphic to H:

$a)$ $G = K_9\,,\quad H = K_{3,2}$

$b)$ $G = K_{8,9}\,,\quad H = P_5$

$c)$ $G = K_{8,9}\,,\quad H = C_5$

$d)$ $G = K_{11}\,,\quad H = K_4 - e$, where e is any edge

$e)$ $G = K_{8,9}\,,\quad H = K_{2,3} - e$, where e is any edge

$f)$ $G = K_{11}\,,\quad H$ is a graph of a 3-dim cube

Problem 2. (15 points = 10 points for part a + 5 points for part b)

a) Find the maximal spanning tree in the weighted graph given by distances in the figure. Compute its weight.

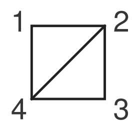
b) Is this maximal spanning tree unique? Explain your answer.

CORRECTION TO FIGURE: Sedona – Prescott distance is 61, not 610.

	Chandler	Flagstaff	Grand Canyon	Lake Havasu	Payson	Phoenix	Prescott	Scottsdale	Sedona	Tucson
Chandler		167	251	222	83	22	122	18	138	98
Flagstaff	167		78	207	115	144	95	150	29	259
Grand Canyon	251	78		232	198	227	127	233	112	343
Lake Havasu	222	207	232		317	202	207	214	231	232
Payson	83	115	198	317		89	100	78	87	191
Phoenix	22	144	227	202	89		100	16	117	116
Prescott	122	95	127	207	100	100		106	61	215
Scottsdale	18	150	233	214	78	16	106		122	120
Sedona	138	29	112	231	87	117	610	122		231
Tucson	98	259	343	232	191	116	215	120	231	

Problem 3. (12 points)

Let $H = K_4 - e$, where $e = (13)$ as in the figure. Compute the number of walks $1 \to 3$ of length 13. If you are using computer assistance, take a screenshot of your computation.



Problem 4. (15 points = 10 points + 5 points)
Suppose a connected planar graph $G = (V, E)$ has score $(3, 3, \ldots, 3)$. Suppose G has only 4-sided, 5-sided and 6-sided faces. It is known that G has four 5-sided faces.

a) Compute the number of 4-sided faces.

b) Is the number of such graphs finite or infinite (up to isomorphism)?

Problem 5. (40 points, 2 points each). **True or False?**

Circle the answers. No reasoning/calculations will be taken into account.

T **F** (1) Every two connected planar graphs with score $(3, \dots, 3)$ are isomorphic

T **F** (2) All planar graphs with score $(4, \dots, 4)$ have an Eulerian circuit

T **F** (3) All complete bipartite graphs $K_{m,n}$ have a Hamiltonian cycle

T **F** (4) All triangulations with the same number of edges have the same number of vertices

T **F** (5) Every tree on 10 vertices has at least 6 bridges

T **F** (6) All triangulations on 10 vertices have no bridges

T **F** (7) All connected graphs on 20 vertices with score $(4, \dots, 4)$ have no bridges

T **F** (8) All trees on 15 vertices with score $(3, \dots, 3, 1, \dots, 1)$ must have the same number of endpoints

T **F** (9) Define a relation $G \bowtie H$ if H is a subgraph of G. Then $\bowtie$ is an equivalence relation

T **F** (10) If H has a Hamiltonian cycle, and H is an induced subgraph of G, then G has a Hamiltonian cycle.

T **F** (11) If H has an Eulerian circuit, and H is an induced subgraph of G, then G has an Eulerian circuit

T **F** (12) Isomorphic graphs have the same number of spanning trees where all vertices have odd degree

T **F** (13) Jordan's curve theorem is used in the proof of non-planarity of $K_{3,3}$

T **F** (14) The proof of Euler's formula given in class uses induction

T **F** (15) The Handshake Lemma is proved by contradiction

T **F** (16) $\log n! = O(n^2)$

T **F** (17) $\binom{n}{2} \sim n^2$

T **F** (18) If a maximal weight spanning tree has the same total weight as a minimal weight spanning tree, then all edges have the same weight

T **F** (19) All complete bipartite graphs with the same score are isomorphic

T **F** (20) Complete graph K_n has at least 3^n spanning trees for all $n \geq 10$