HOMEWORK 7 (MATH 180, WINTER 2021)

Read: **MN** (Second ed.), section 8.5

Solve: Exercises in MN:

Solve 3 in §8.5 using two methods:
  a) Prüfer's code (modified appropriately), and
  b) Matrix tree theorem

Additional exercises:

I. Let $G = (V, E)$ be a connected simple graph, and let Q_G be the $n \times n$ matrix defined in class in the construction of the matrix tree theorem. Denote by $\lambda_1, \ldots, \lambda_n$ the eigenvalues of Q_G.
  a) Prove that one of the eigenvalues $\lambda_1 = 0$.
  b) Compute $\lambda_2 \cdots \lambda_n$
  c) Compute $\lambda_2 + \ldots + \lambda_n$

II. Let $G = (V, E)$ be a connected simple graph, $|V| = n$. Graph $\overline{G} = (V, \overline{E})$ is called a *complement graph*, if set $\overline{E}$ is the complement to E. For example, $\overline{O_n} = K_n$ and $\overline{K_{a,b}} = K_a \cup K_b$.

a) Find a simple graph G, such that $\overline{G}$ is isomorphic to G. Is the number of such graphs (up to isomorphism) finite or infinite?

b) Let G, H be simple graphs on n vertices with the same number of spanning trees. Prove or disprove: graphs $\overline{G}$ and $\overline{H}$ have the same number of spanning trees.

c) Prove that if G is planar then $\overline{G}$ is not planar, for all $n \geq 10$.

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This Homework is due Wednesday March 3, at 8:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic. The explanation can be brief, but must indicate all logical steps.

P.S. Each item above has the same weight.