

HOMEWORK 4 (MATH 180, WINTER 2021)

Read: MN (Second ed.), sections 5.1, 5.3 and 5.4.

Solve: Exercises in MN:

4, 5 in §5.1

2, 4 in §5.4

Additional exercises:

I. Suppose tree T has 20 vertices of degree 3, one vertex of degree 10 and all other vertices are endpoints. Find the number of vertices in T .

II. Let $G = (V, E)$ be a simple graph on $n = |V|$ vertices. Suppose the distance between two vertices $v, w \in V$ is equal to $n - 1$. Prove that $G \simeq P_n$ and that v, w are endpoints.

III. Find all n , such that $K_{5,n}$ has a Hamiltonian cycle.

IV. (a) Let e be an edge in a complete graph K_{20} . Find the probability that a random spanning tree in K_{20} contains e .

(b) Same question for a complete bipartite graph $K_{10,10}$.

V. Consider the table of distances between British cities in Figure 1. For example, the distance from Manchester to Coventry is 160. Use the table to define weights on edges of K_8 . Find the minimal and maximal spanning tree in that graph.

	Birmingham						
177							
32	193						
67	117	66					
146	55	149	80				
172	16	186	111	419			
129	57	160	93	442	67		
119	59	126	57	376	54	63	
							Sheffield

FIGURE 1. Distances between selected British cities.

VI. Find a maximal spanning tree T in the large graph in Figure 2. Is this maximal spanning tree unique? What is the total sum of weights on the unique path between the green and red vertices in T ?

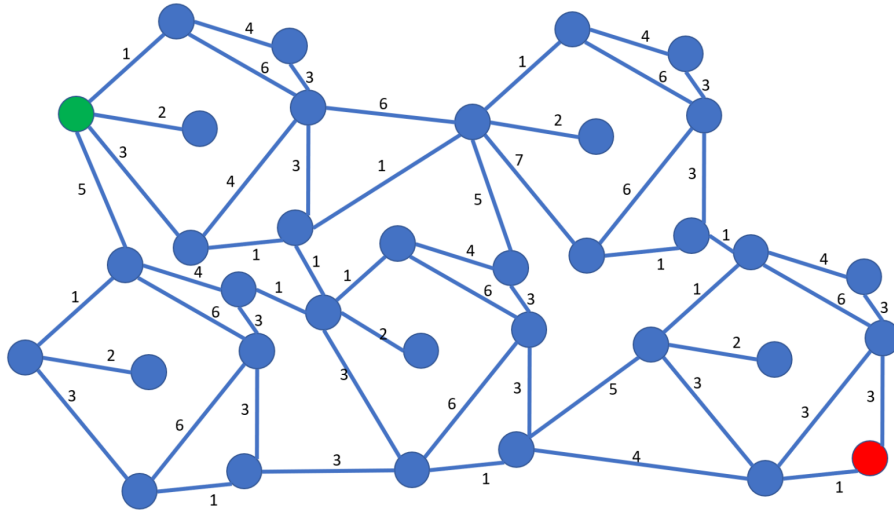


FIGURE 2. Large weighted graph.

This Homework is due Wednesday Feb 3, at 8:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic. The explanation can be brief, but must indicate all logical steps.

P.S. Each item above has the same weight.