

HOMEWORK 3 (MATH 180, WINTER 2021)

Read: MN (Second ed.), sections 4.2, 4.4.

Solve: Exercises in MN:

8, 9bc, 12 in §4.2

2, 3, 7, 10ab in §4.4

Additional exercises:

- I.** *a)* Find a connected graph with score $(4, 4, 4, 4, 4, 4, 4, 4, 4, 4)$ which has a Hamiltonian cycle.
b) Find a connected graph with score $(4, \dots, 4)$ which does not have a Hamiltonian cycle (you can use any number of vertices of degree 4).
- II.** Consider a graph $G = (V, E)$, with $V = \{1, 2\}$ and $E = \{(1, 1), (1, 2)\}$, with one loop and one edge. Prove that the number of walks $1 \rightarrow 1$ of length ℓ is the Fibonacci number F_ℓ .
- III.** Let P_n be a n -path on vertices $\{1, \dots, n\}$, where $n > 100$. Compute the number of walks $1 \rightarrow n$
- a)* of length $n - 1$,
 - b)* of length n ,
 - c)* of length $n + 1$,
 - d)* of length $n + 3$.
- IV.** Let $K_{3,3}$ be a complete bipartite graph on vertices $\{1, 2, 3\}$ in one part and vertices $\{4, 5, 6\}$ in the other part. Compute the number of walks $1 \rightarrow 1$
- a)* of length 20,
 - b)* of length 8, and which avoid edge $(1, 4)$.

This Homework is due Wednesday Jan 27, at 8:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic. The explanation can be brief, but must indicate all logical steps.

P.S. Each item above has the same weight.